Understanding Number System

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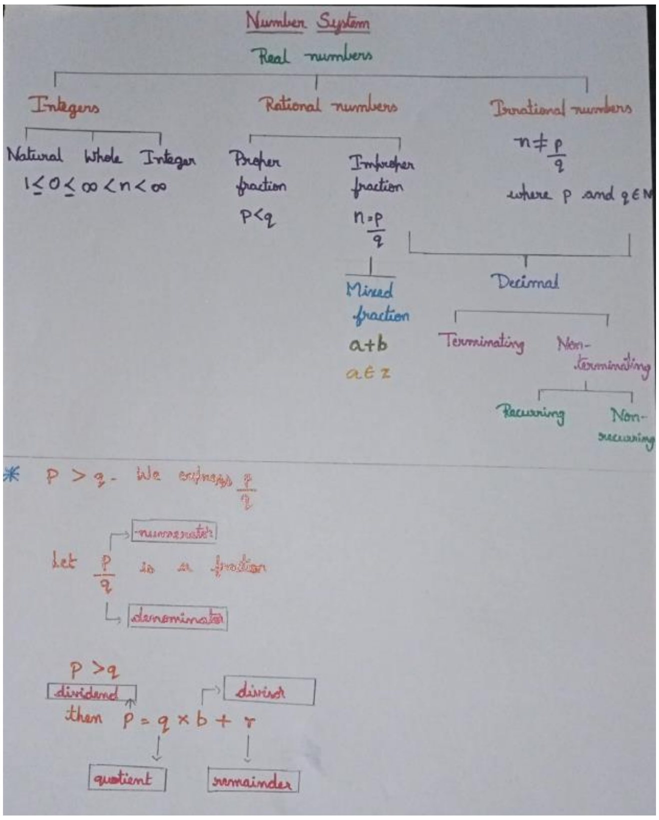


Number System

Real numbers

Integers — Natural, Whole, Integer

$1 \le 0 \le \infty < n < \infty$

Rational numbers — Proper fraction $p < q$; Improper fraction $n = \frac{p}{q}$

Mixed fraction $a + b$, $a \in Z$

Irrational numbers $n \ne \frac{p}{q}$ where p and $q \in N$

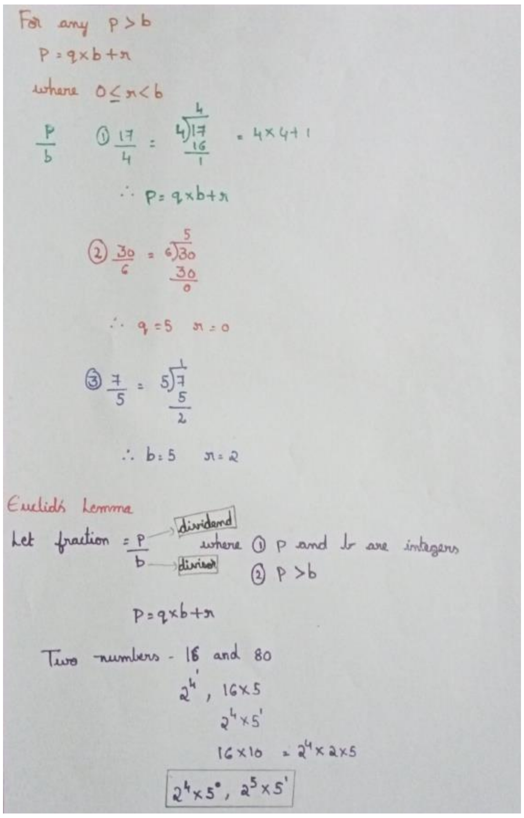
Decimal — Terminating; Non-terminating — Recurring, Non-recurring

* $p > q$. We express $\frac{p}{q}$

let $\frac{p}{q}$ is a fraction — numerator, denominator

$p > q$ — dividend, divisor

then $p = q \times b + r$ — quotient, remainder



For any $p > b$

$p = q \times b + r$

where $0 \le r < b$

$\frac{p}{b}$ ① $\frac{17}{4} = 4\overline{)17}$, 16, 1 $= 4 \times 4 + 1$

$\therefore p = q \times b + r$

② $\frac{30}{6} = 6\overline{)30}$, 30, 0

$\therefore q = 5 \quad r = 0$

③ $\frac{7}{5} = 5\overline{)7}$, 5, 2

$\therefore b = 5 \quad r = 2$

Euclid's Lemma

Let fraction $= \frac{p}{b}$ — dividend, divisor — where ① p and b are integers ② $p > b$

$p = q \times b + r$

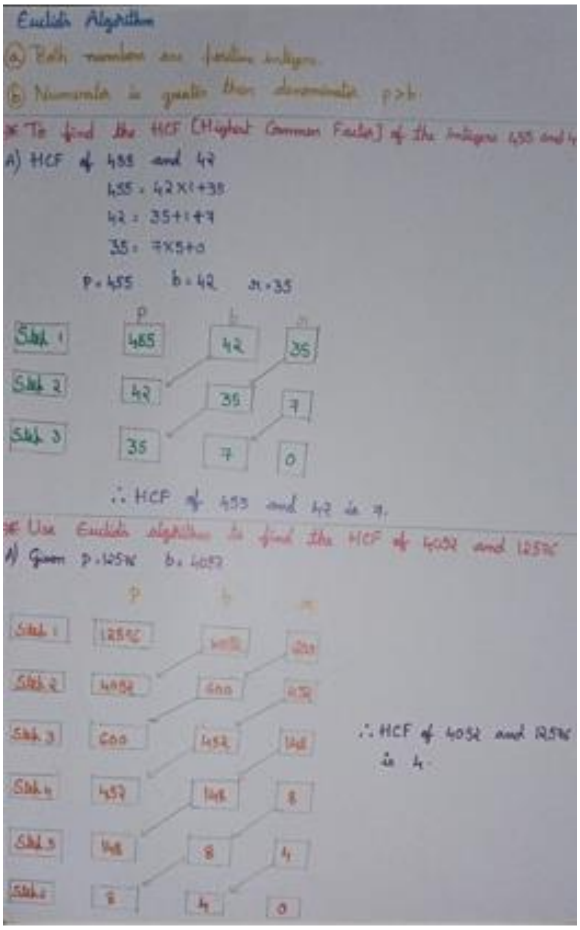
Two numbers - 16 and 80

2^4, 16×5

$2^4 \times 5^1$

$16 \times 10 = 2^4 \times 2 \times 5$

$\boxed{2^4 \times 5^0,\ 2^5 \times 5^1}$



Euclid's Algorithm

ⓐ Both numbers are positive integers

ⓑ Numerator is greater than denominator $p > b$

* To find the HCF (Highest Common Factor) of the integers 455 and 42

A) HCF of 455 and 42

$455 = 42 \times 10 + 35$

$42 = 35 \times 1 + 7$

$35 = 7 \times 5 + 0$

$p = 455 \quad b = 42 \quad r = 35$

	p	b	r
Step 1	455	42	35
Step 2	42	35	7
Step 3	35	7	0

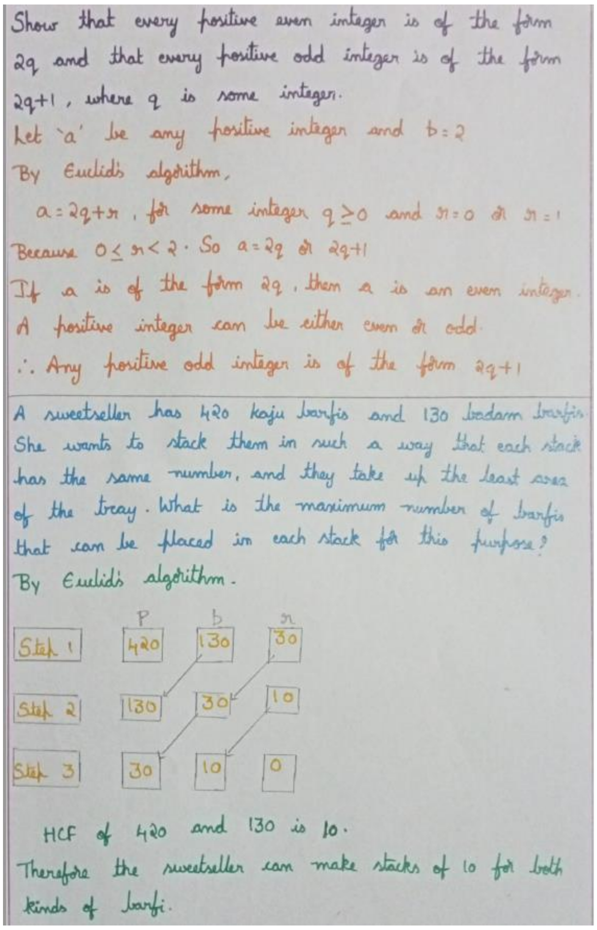
$\therefore$ HCF of 455 and 42 is 7.

* Use Euclid's algorithm to find the HCF of 4052 and 12576

A) Given $p = 12576$ $b = 4052$

	p	b	r
Step 1	12576	4052	420
Step 2	4052	420	272
Step 3	420	272	148
Step 4	272	148	124
Step 5	148	124	24
Step 6	124	24	4

$\therefore$ HCF of 4052 and 12576 is 4.



Show that every positive even integer is of the form $2q$ and that every positive odd integer is of the form $2q+1$, where q is some integer.

Let 'a' be any positive integer and $b = 2$

By Euclid's algorithm,

$a = 2q + r$, for some integer $q \ge 0$ and $r = 0$ or $r = 1$

Because $0 \le r < 2$. So $a = 2q$ or $2q+1$

If a is of the form $2q$, then a is an even integer.

A positive integer can be either even or odd

$\therefore$ Any positive odd integer is of the form $2q+1$

A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of barfis that can be placed in each stack for this purpose?

By Euclid's algorithm.

	p	b	r
Step 1	420	130	30
Step 2	130	30	10
Step 3	30	10	0

HCF of 420 and 130 is 10.

Therefore the sweetseller can make stacks of 10 for both kinds of barfi.

* Take two numbers 75 and 150

$5^2 \times 3^1 \times 2^0 \cdot 5^2 \times 3^1 \times 2^1$

HCF $= 5^2 \times 3^1 \times 2^0$

$= 25 \times 3$

$= 75$

- HCF is the product of prime factors of given number with their lowest power.
- Lowest power of prime factor.

5 - lowest power $a = 2$

3 - lowest power $b = 1$

2 - lowest power $c = 0$

* To prove that $\sqrt{2}$ is an irrational number.

A) Assume $\sqrt{2} = \frac{p}{q}$ where p and q are co-primes i.e, no common factor.

Squaring on both sides $(p)^2 = \sqrt{2}q$

$p^2 = 2q^2$

Let 'p' has a factor 'm'.

$p^2 = m \times m$ p^2 also has factor 'm'.

$p^2 = 2q^2$ 2 is a factor of p

$\therefore$ q also has a factor 2

2 is a common factor between p and q

$\frac{p}{q}$ is not a rational number.

This is in contradiction to assumption made in the beginning that p and q are co-primes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

So, we conclude that $\sqrt{2}$ is irrational

Let x be a rational number whose decimal expansion terminates then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.

A) $x = 3.56 = \frac{p}{q} = \frac{3.56}{100} = \frac{356}{100}$

Here p and q are -
1. co-primes
2. $b = 2^0 \times 5^2$

$m = 0$ $n = 2$

Both are integers

$= \frac{356}{(10)^2} = \frac{356}{(2 \times 5)^2} = \frac{356}{2^2 \times 5^2}$

$= \frac{\cancel{356}^{\,178\ 89}}{\cancel{2} \times \cancel{2} \times 5^2} = \frac{89}{25}$

Let's use Euclid's division algorithm

	p	b	r
Step 1	89	25	14
Step 2	25	14	11
Step 3	14	11	3
Step 4	11	3	2
Step 5	3	2	1
Step 6	2	1	0

$\therefore$ HCF is '1'.

Problems -

Using Euclid's division algorithm, check whether 231, 396 are co-prime or not justify your answer.

q - 231 and p - 396 $\frac{p}{q}$

	p	b	r
Step 1	396	231	165
Step 2	231	165	66
Step 3	165	66	33
Step 4	66	33	0

33 is common factor

$396 = 33 \times 12$

$231 = 33 \times 7$

Since 33 is a common factor of 396 and 231

Hence they are not co-primes.

Give examples of polynomials $p(x)$, $g(x)$ and $r(x)$, $q(x)$ which satisfy the division algorithm

i) deg $p(x)$ = deg $q(x)$ ii) deg $q(x)$ = deg $r(x)$ iii) deg $r(x) = 0$

1) deg $p(x)$ = deg $q(x)$

$P(x) = g(x)\ q(x) + r(x)$

Degree $x^3 + x^2 + 9x + 7$ Degree of this polynomial is 3

$P(x) = (x^2 + x)(x^3 + x^2 + 7)$

deg of $P(x)$ = deg of $f_1(x)$ + deg of $f_2(x)$

$2 + 3 = 5$

Let $q(x) = x^3 + 2x^2 + 7$

$P(x) = 5 \times (x^3 + 2x^2 + 7)$

In Euclid's lemma

$\frac{P(x)}{g(x)}$ then we can write

$P(x) = g(x)\ q(x) + r(x)$

degree of $r(x)$ is one less than degree of $g(x)$

(degree of $q(x) = m = x + 2$)

$x + 2 - q(x) = m$

$g(x)$ $x^2 + 1 \overline{) x^3 + 2x^2 + 5x + 7}$

degree of $g(x) = 1 = 2$

$x^3 + x$

$2x^2 + 4x$

$2x^2 + 2$

$4x - 2$ $r(x)$ - degree of $r(x) = n = 1$

degree of $P(x)$ = degree of $g(x)$ + degree of $q(x)$

degree of $r(x)$ = degree of $g(x) - 1$

ii) deg $q(x)$ = deg $r(x)$

$K = L + M$

$K = L + M$

$M = L - 1$

$q(x) = 2x^3 + 3x + 5$

$M = 3$

$P(x) = 7(2x^3 + 3x + 5)$

$M = M$

$K = L + M$

$= (M + 1)$

$q(x) = 5x^3 + 3x^2 - 5$ - $M = 3$

$r(x) = 4x^3 + 2x$ - $N = 3$

$g(x) = 7x^4 + 5x^3 - 2x^2 + 3$ - $L = M + 1 = 4$

$P(x) = g(x) \times q(x) + r(x)$

$= (7x^4 + 5x^3 - 2x^2 + 3) \times (5x^3 + 3x^2 - 5) + (4x^3 + 2x)$

$= (7x^4 + 25x^3 - 6x^2 - 15) + (4x^3 + 2x)$

$= 7x^4 + 29x^3 - 6x^2 + 2x - 15$

K - degree of $P(x)$

L - degree of $g(x)$

M - degree of $q(x)$

N - degree of $r(x)$

$K = L + M$

$M = L - 1$

$L = M + 1$

iii) deg $r(x) = 0$ i.e., $N = 0$

$r(x) = 15$

$q(x) = x^5 \cdot x^3 + x^2 - 5$

$g(x) = 5x^3 - 8x^2 + 2$

$P(x) = g(x) + q(x) + r(x)$

$= (5x^3 - 8x^2 + 2) + (x^5 - x^3 + x^2 - 5) + 15$

$= x^5 + 4x^3 - 7x^2 - 3 + 15$

$\frac{P(x)}{g(x)}$ then

divisor

$P(x) = g(x) \times q(x) + r(x)$

dividend

quotient

remainder

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