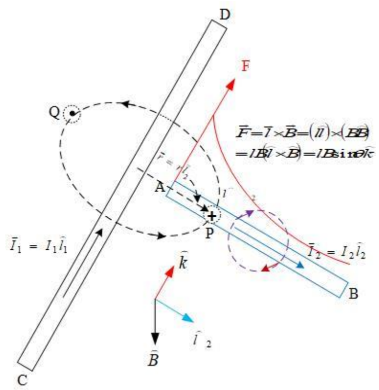
*Code: Phy/ EM-4/MEC/001*

# Electromagnetism: Magnetic Effect of Electric Current

## Representative Questions (Set-1)

<table>
<tr><td>Q-1</td><td>Two wires carrying equal currents $i$ each, are placed perpendicular to each other, just avoiding a contact. If one wire is held fixed and the other is free to move under magnetic forces, what kind of motion will result?</td></tr>
<tr><td>A-1</td><td>Movable conductor would align parallel to fixed conductor and get repelled from it.</td></tr>
<tr><td>I-1</td><td>

Given are two wires AB and CD carrying currents $\vec{I}_2 = I_2\hat{l}_2$ and $\vec{I}_1 = I_1\hat{l}_1$ respectively. Here, $I_1$ and $I_2$ are magnitudes of the currents and $\hat{l}_1$ and $\hat{l}_2$ are direction vectors of the two currents and are along the length of the two conductors.




Given that the two conductors are placed perpendicular to each other, it is visualized to be horizontally placed on the table top shown in the figure. Further, magnitudes of the two currents are equal i.e. $I_1 = I_2 = i$…(1); conductor carrying current $I_1$ is kept fixed while the conductor AB carrying current $I_2$ is free to move.

Here, Two electromagnetic phenomenon come into play as under-

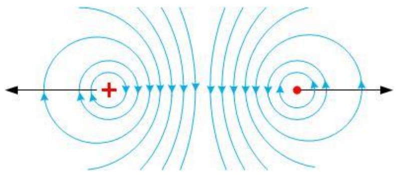
**Biot-Savart's Law:** Produces magnetic field $\vec{B} = B\hat{B} = \frac{\mu_0 i}{2\pi r}\hat{B}$…(2), around the conductor. Direction of the magnetic field, as per Ampere's Right-Hand Thumb Rule, is piercing the table top though the conductor current carrying current $I_2$ and emerging out of the table top from diametrically opposite point.

**Lorentz's Force:** Magnetic flux $\vec{B}$ produced by conductor carrying CD current $I_1$ when interacts with current $I_2$ the conductor AB would experience a force $\vec{F} = I_2 B \sin 90\,\hat{k} \Rightarrow \vec{F} = \frac{\mu_0 i^2}{2\pi r}\hat{k}$ …(3), at each point. Here, $\hat{k}$ is perpendicular to the plane of unit vectors $\hat{B}$ and $\hat{l}_2$, as shown in the figure.

Further, magnitude of $\vec{B} \propto \frac{1}{r}$ hence force at end A would be larger than that end B. Thus, non-uniform force on the conductor AB carrying would create two kinds of motion, D' Alembert's Principle, as per mechanics –

- **a.** Translational acceleration in the direction of force as per newton's Second Law of motion $a = \frac{F_R}{m}$, here $F_R = \int_a^b F dr$ and is $m$ mass of the rod.
- **b.** In this case non-uniform distribution of force $F$ along the conductor, as shown through the graph in the figure, produces a torque. This torque produces a rotational motion in the conductor AB, about the center of mass. The direction of the motion is depicted through a circle around the center of mass of uniform conductor AB.

As a result of the mechanics involved, end A will tend to move towards D and end B would tend to move towards end C, as shown in the figure. Eventually the conductor AB would tend align parallel to the conductor CD.



Once the two conductors are in parallel, currents through them would be in opposite directions leading to force of repulsion, causing the conductor AB to move away from the fixed conductor CD, as shown in the figure.

**N.B.: 1.** The understanding of the figure (3D system on 2D) requires visualization; basic knowledge of drawings is helpful for this.

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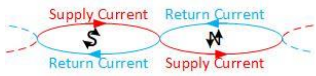
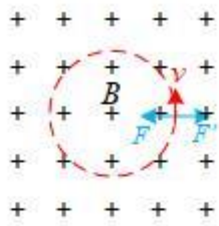
| | **2.** This problems involves integration of multiple concepts, both in electromagnetism and mechanics, and thus it is of a high quality. |
| Q-2 | Quite often, connecting wires carrying currents in opposite directions are twisted together in using electrical appliances. Explain how it avoids unwanted magnetic field. |
| A-2 | Does not allow unidirectional magnetic field in untwisted connecting wires. |
| I-2 | Twisting of electrical wires used in electrical appliances creates small loops as shown in the figure. Characteristically electric current produces magnetic field. Thus twists in connecting wires produce small loops as shown in the figure.  <br><br>In case there are no twists the current loop formed by connecting wires will be large and magnetic field produced by it will cause stray effects on adjoining equipment.<br><br>But in case twists, small current loops formed by the connecting wires polarity of magnetic field in consecutive loops is opposite and therefore it closes in every consecutive loop.<br><br>**N.B.:** This problem is a good example of practical application. Students are advised to verify magnetic effect of current in connecting wires, using a magnetic compass, supplying electricity from battery or a AC/DC converter, both twisted and untwisted. |
| Q-3 | Two particles X and Y having equal charge, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular path of radii $R_x$ and $R_y$ respectively. The ratio of the mass of X and that of Y is –<br><br>(a) $\left(\frac{R_x}{R_y}\right)^{\frac{1}{2}}$ (b) $\frac{R_y}{R_y}$ (c) $\left(\frac{R_x}{R_y}\right)^2$ (d) $R_xR_y$ |
| A-3 | (c) |
| I-3 | Given that particles X and Y have equal charge say $q$ and are accelerated through equal potential difference say $V$. Then potential energy of the particles after acceleration shall also be equal $E = qV$…(1)<br><br>Let mass of the two particles be $m_x$ and $m_y$. Then velocity attained by the particles will be $\frac{1}{2}mv^2 = E \Rightarrow \frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$….(2) Accordingly, velocity of particles X and Y is would be $v_x = \sqrt{\frac{2qV}{m_x}}$ and $v_y = \sqrt{\frac{2qV}{m_y}}$…(3), respectively.<br><br>The forces on the particles inside magnetic field describe a circular motion in a plane perpendicular to the magnetic field, maintaining an equilibrium. Accordingly, kinetic energy remains at $E$.  <br><br>It is important to note that electromagnetic force on particle X is $\vec{F_x} = qv_xB\hat{r}$ and on particle Y is $\vec{F_y} = qv_yB\hat{r}$. These forces are perpendicular to the plane of vectors $\vec{v}$ and $\vec{B}$ and act as centripetal force. Accordingly, centrifugal forces are $\overrightarrow{F_x'} = \frac{mv_x{}^2}{r_x}(-\hat{r})$ and $\overrightarrow{F_y'} = \frac{mv_y{}^2}{r_y}(-\hat{r})$, respectively.<br><br>Necessary condition of circular motion of particles X is $\vec{F_x} + \overrightarrow{F_x'} = 0 \Rightarrow qv_xB = \frac{m_xv_x{}^2}{r_x} \Rightarrow m_x = \frac{qBr_x}{v_x}$. Likewise, for particle Y it is $\vec{F_y} + \overrightarrow{F_y'} = 0 \Rightarrow qv_yB = \frac{m_yv_y{}^2}{r_y} \Rightarrow m_y = \frac{qBr_y}{v_y}$…(4). |

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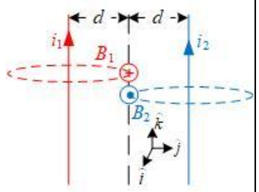
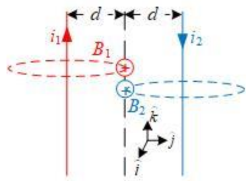
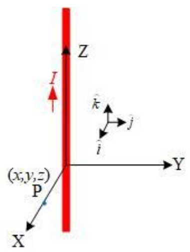
| | Combining set of equations (3) and (4) for the particle X is $m_x = \frac{qBr_x}{\sqrt{\frac{2qV}{m_x}}} \Rightarrow \sqrt{m_x} = \frac{\sqrt{q}\times Br_x}{\sqrt{2V}}$. Likewise for particle Y it is $m_y = \frac{qBr_y}{\sqrt{\frac{2qV}{m_y}}} \Rightarrow \sqrt{m_y} = \frac{\sqrt{q}\times Br_y}{\sqrt{2V}}$. Accordingly, $\frac{\sqrt{m_x}}{\sqrt{m_y}} = \frac{\frac{\sqrt{q}\times Br_x}{\sqrt{2V}}}{\frac{\sqrt{q}\times Br_y}{\sqrt{2V}}} \Rightarrow \sqrt{\frac{m_x}{m_y}} = \frac{r_x}{r_y} \Rightarrow \frac{m_x}{m_y} = \left(\frac{R_x}{R_y}\right)^2$, as provided in **option (c), is the answer**. |
| Q-4 | Two parallel long wires carry currents $i_1$ and $i_2$ with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of $i_2$ is reversed, the field becomes 30 μT. The value of $\frac{i_1}{i_2}$ is –<br>(a) 4 (b) 3 (c) 2 (d) 1 |
| A-4 | (c) |
| I-4 | Magnitude of the magnetic field at point at a distance $d$ from a conductor carrying current $i$ is $B = \frac{\mu_0 i}{2\pi d}$. In the problem there are two conductors carrying currents $i_1$and $i_2$ such that $i_1 > i_2$. Therefore, at a point between the two conductors, magnetic field due to currents $i_1$and $i_2$ as per Biot-Savart's Law is $B_1 = \frac{\mu_0 i_1}{2\pi d}$ and $B_2 = \frac{\mu_0 i_2}{2\pi d}$, respectively.<br><br><br>When both the currents $i_1$and $i_2$ are in same direction then direction of the magnetic fields $B_1$and $B_2$, as per Ampere's Right Hand Thumb Rule are opposed to each other. Therefore net magnetic field at midway is $B = B_1 - B_2 \Rightarrow B = \frac{\mu_0 i_1}{2\pi d} - \frac{\mu_0 i_2}{2\pi d} = 10$ T. It leads to $\frac{\mu_0}{2\pi d}(i_1 - i_2) = 10$ T…(1).<br><br><br>But, when currents in the two conductors in opposite direction, magnetic field at midway of the two conductors are in same direction. Hence, is $B' = B_1 - B_2$. It leads to $B' = \frac{\mu_0 i_1}{2\pi d} + \frac{\mu_0 i_2}{2\pi d} = 30 \Rightarrow \frac{\mu_0}{2\pi d}(i_1 + i_2) = 30$ T…(2).<br>Using (1) and (2), $\frac{\frac{\mu_0}{2\pi d}(i_1+i_2)}{\frac{\mu_0}{2\pi d}(i_1-i_2)} = \frac{30}{10} \Rightarrow \frac{i_1+i_2}{i_1-i_2} = \frac{3}{1}$…(3). Applying componendo-dividendo, of ratio & proportions to (3) we have $\frac{(i_1+i_2)+(i_1-i_2)}{(i_1+i_2)-(i_1-i_2)} = \frac{3+1}{3-1} \Rightarrow \frac{i_1}{i_2} = 2$ as provided in **option (2), is the answer**. |
| Q-5 | A current of 10 A is established in a long wire along the positive Z-axis. Find the magnetic field $\vec{B}$ at the point (1m,0,0). |
| A-5 | $2 \times 10^{-6}$T along Y-axis |
| I-5 | It is required to determine magnetic field at point P whose position vector, as per given data, shown in the diagram is $\vec{r} = 1\hat{\imath} + 0\hat{\jmath} + 0\hat{k} \Rightarrow \vec{r} = r\hat{\imath} = 1\hat{\imath} \Rightarrow r = 1$…(1)Current through a long straight wire is $\vec{I} = I\hat{k} = 10\hat{k} \Rightarrow I = 10$…(2) It is required to find magnetic field vector at P. As per Biot-Savart's Law magnetic field $\vec{B} = \frac{\mu_0 \vec{I}\times\vec{r}}{2\pi r^2}$…(3).<br><br><br>Combining (1), (2) and (3) with that $\frac{\mu_0}{4\pi} = 10^{-7}$ we have, $\vec{B} = \frac{\mu_0(10\hat{k})\times(1\hat{\imath})}{2\pi 1^2}$T. It leads to $\vec{B} = 2\left(\frac{\mu_0}{4\pi}\right) \times 10 \times (\hat{k} \times \hat{\imath}) \Rightarrow \vec{B} = (2(10^{-7}) \times 10)\hat{\jmath} \Rightarrow \vec{B} = 2 \times 10^{-6}\hat{\jmath}$ T. Accordingly, flux density $\vec{B}$ at P is of magnitude **$2 \times 10^{-6}$T along Y-axis, is the answer.**<br>**N.B.:** It is to be noted that mathematical operator between two scalars is simple multiplication and it assumes vector product only when sandwiched between two vector quantities viz. $(\hat{k} \times \hat{\imath}) = \hat{\jmath}$, while $(\hat{\imath} \times \hat{k}) = (-\hat{\jmath})$. |

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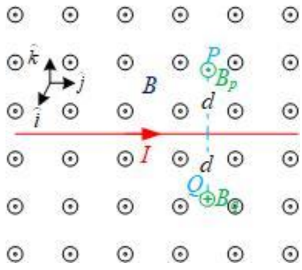
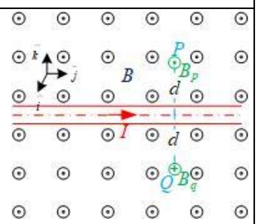
| Q-6 | A long straight wire carries a current 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-5}$ T pointing vertically upward as shown in the figure. Find the magnitude of the resultant magnetic field at the points P and Q, both situated at a distance of 2.0 cm from the wire in the same horizontal plane. |
| A-6 | |
| I-6 | <br>Given system is shown in figure in horizontal plane $\hat{\imath} - \hat{\jmath}$ . A conductor is carrying current $i$ of value $\hat{I} = 1.0\hat{\jmath}$ A. It is placed in a vertical magnetic field $B$ of value $\vec{B} = 1.0 \times 10^{-5}\hat{k}$T…(1) It is required to determine resultant magnetic field at point P and Q at a distance $d = 2.0 \times 10^{-2}$m and their position vectors are defined as $\vec{r}_p = 2.0 \times 10^{-2}(-\hat{\jmath})$m and $\vec{r}_q = 2.0 \times 10^{-2}(\hat{\jmath})$m …(2), respectively.<br><br>Magnetic field vector at a point due to long strait current carrying wire, as per Biot-Savart's Law, is $\vec{B}' = \frac{\mu_0 \vec{I} \times \vec{r}}{2\pi r^2} \Rightarrow \vec{B}' = \frac{\mu_0 I}{2\pi r}\hat{I} \times \hat{r}$. With given orientation of conductor and symmetrical position of points P and Q $\vec{B}_p{}' = \frac{\mu_0 I}{2\pi r}\hat{\jmath} \times (-\hat{\imath}) \Rightarrow \vec{B}_p{}' = \frac{\mu_0 I}{2\pi r}\hat{k}$ and $\vec{B}_q{}' = \frac{\mu_0 I}{2\pi r}\hat{\jmath} \times \hat{\imath} \Rightarrow \vec{B}_q{}' = \frac{\mu_0 I}{2\pi r}(-\hat{k})$….(3).<br><br>The final form in (3) is based on vector product, again using Right-Hand Thumb Rule in different context, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$ and $\hat{\jmath} \times (-\hat{\imath}) = \hat{k}$.<br><br>Combining (1), (2) and (3), resultant magnetic field at P is $\vec{B}_p = \vec{B} + \vec{B}_p{}' \Rightarrow \vec{B}_p = B\hat{k} + \frac{\mu_0 I}{2\pi r}\hat{k}$. It leads to $\vec{B}_p = \left(B + \frac{\mu_0 I}{2\pi r}\right)\hat{k}$. Likewise, at Q it is $\vec{B}_q = \vec{B} + \vec{B}_q{}' \Rightarrow \vec{B}_q = B\hat{k} + \frac{\mu_0 I}{2\pi r}(-\hat{k})$.<br><br>It leads to $\vec{B}_q = \left(B - \frac{\mu_0 I}{2\pi r}\right)\hat{k}$.<br><br>Using the available data, $\vec{B}_p = (1.0 \times 10^{-5})\hat{k} + 2 \times 10^{-7}\frac{1.0}{2.0\times10^{-2}}\hat{k} = (1.0 \times 10^{-5} + 1.0 \times 10^{-5})\hat{k} \Rightarrow \vec{B}_p = 2.0 \times 10^{-5}\hat{k}$, and $\vec{B}_q = (1.0 \times 10^{-5})\hat{k} + 2 \times 10^{-7}\frac{1.0}{2.0\times10^{-2}}\hat{k} = (1.0 \times 10^{-5} - 1.0 \times 10^{-5})\hat{k} \Rightarrow \vec{B}_q = 0$<br><br>**N.B.: 1.** The problem can be easily solved applying Ampere's Right-Hand Thumb Rule. Yet, at first stage of practicing the problem-solving, it is illustrated using vector equation as per Biot-Savart's Law. Apparently, it is more tedious, yet it will help to develop visualization and conceptual clarity. As student's go into proficiency and revision stage they can comfortably and confidentially resort to short-cuts.<br><br>**2**. For zero quantity has neither direction nor unit. |
| Q-7 | A long straight wire of radius $R$ carries a current $I$ and is placed horizontally in a magnetic field $B$ pointing vertically upward. The current is uniformly distributed over its cross-section.<br>(a) At what points will the resultant magnetic field have maximum magnitude? What will be the maximum magnitude?<br>(b) What will be the minimum magnitude of the resultant magnetic field? |
| A-7 | (a) $B + \frac{\mu_0 I}{2\pi R}$ and (b) $B - \frac{\mu_0 I}{2\pi R}$. |
| I-7 | <br>Given system is shown in figure in horizontal plane $\hat{\imath} - \hat{\jmath}$ . A conductor is carrying current $I$ of value $\hat{I} = I\hat{\jmath}$ A. The current is uniformly distributed over the cross-section of the conductor, the current density is $\rho = \frac{I}{\pi R^2}$. It is placed in a vertical magnetic field $B$ of value $\vec{B} = B\hat{k}$…(1) It is required to determine resultant magnetic field at point P and Q at a distance $d$ and their position vectors are defined as $\vec{r}_p = d(-\hat{\jmath})$m and $\vec{r}_q = d\hat{\jmath}$ m …(2), respectively. |

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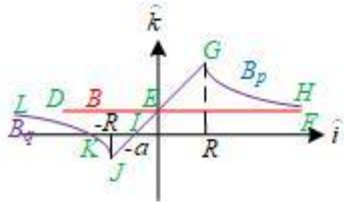
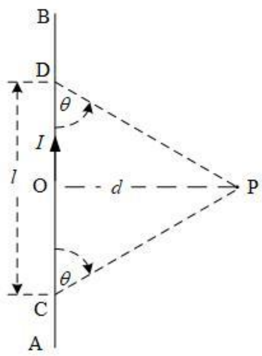
| | In this case magnetic field produced by the conductor interacts with magnetic field $B$ around it. It has following aspects-<br><br><br>(i) Magnetic field above the axis of the conductor is along $(\hat{k})$ and is additive to the magnetic field B and is shown in the graph by curve sections EG and GH, already brought out in the figure above.<br>(ii) Magnetic field below the axis of the conductor is along $(-\hat{k})$ and is subtractive to the magnetic field B and is shown in the graph by curve sections EIJ and JKL, already brought out in the figure above.<br>(iii) Inside the conductor $0 \leq r < R$ magnetic field due current in it, as per Biot-Savart's Law, combined with Ampere's Circuital Law, is $B_c = \frac{\mu_0 I r}{2\pi R^2}$ and is shown in the graph by curve sections EG, above the axis of the conductor and EJ, below the axis of the conductor.<br>(iv) Outside the conductor $R \leq r < \infty$ magnetic field due current in it, as per Biot-Savart's Law, is $B_c = \frac{\mu_0 I}{2\pi r}$ and is shown in the graph by curve sections GH, above the axis of the conductor and JL, below the axis of the conductor<br>With this analysis we are ready find answer to the two parts of the question-<br>**Part (a):** It is seen from the graph of resultant magnetic field that maximum magnetic field occurs at the surface of the conductor where $r \to r_p \Rightarrow r = R$, above the axis of the conductor as discussed at (i) and (iii) above. Accordingly magnitude of the maximum magnetic field is $B_{max} = B + \frac{\mu_0 I}{2\pi R}$, is answer of part (a).<br>**Part (a):** Again observation of graph of resultant magnetic field, below the axis of the conductor inside the conductor it passes through Zero value at $r_q \to r \Rightarrow r = -a$. But, it attains minimum value at J on the graph when $r = -R$ and again starts increasing. Therefore, minimum flux density at occurs at $r = -R$ is $B_{min} = B - \frac{\mu_0 I}{2\pi R}$, is the answer of part (b).<br>Thus, **answers are (a)** $\boldsymbol{B + \frac{\mu_0 I}{2\pi R}}$ **and (b)** $\boldsymbol{B - \frac{\mu_0 I}{2\pi R}}$**.**<br>**N.B.:** (a) This is an excellent problem on interaction of magnetic fields.<br>(b) Reference to the graph must be drawn from $\hat{\imath} - \hat{\jmath} - \hat{k}$ space shown in the figure.<br>(c) While determining minimum value concept of number like is important. |
| Q-8 | A long straight wire carries a current $I$. Let $B_1$ be the magnetic field at a point P at a distance $d$ from the wire. Consider a section of length $l$ of this wire such that the point P lies on a perpendicular bisector of the section. Let $B_2$ be the magnetic field at this point due to this section only. Find the value of $\frac{d}{l}$ so that $B_2$ differs from $B_1$ by 1%. |
| A-8 | 0.07 |
| I-8 | Given long straight wire AB carrying current $I$ is shown along with a point P at a distance $d$ from the wire. Magnetic field at the point as per Biot-Savart's Law is $B_1 = \frac{\mu_0 I}{2\pi d}$…(1) A section of wire CD of length $l$ is so located, as given, that angle between position vector and the wire is $\theta$. In this case magnetic field with the application of Biot-Savart's Law is $B_2 = \frac{\mu_0 I}{2\pi d}\cos\theta$…(2), here trigonometrically $\cos\theta = \frac{\frac{l}{2}}{r}$ where distance CP is $r = \sqrt{d^2 + \left(\frac{l}{2}\right)^2}$. Accordingly, $\cos\theta = \frac{\frac{l}{2}}{\sqrt{d^2+\left(\frac{l}{2}\right)^2}} \Rightarrow \cos\theta = \frac{l}{\sqrt{4d^2+l^2}}$...(3)<br><br> |

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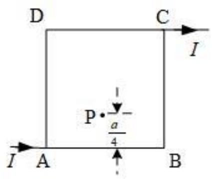
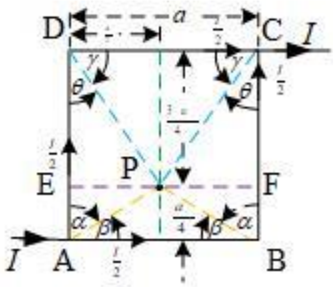
| | Given that $\frac{B_1-B_2}{B_1}=\frac{1}{100}$. Combining (1), (2) and (3) $\frac{\frac{\mu_0 I}{2\pi d}-\frac{\mu_0 I}{2\pi d}\times\frac{l}{\sqrt{4d^2+l^2}}}{\frac{\mu_0 I}{2\pi d}}=\frac{1}{100}$. It simplifies into $\frac{\sqrt{4d^2+l^2}-l}{\sqrt{4d^2+l^2}}=\frac{1}{100}$.<br>Applying dividendo $\frac{l}{\sqrt{4d^2+l^2}}=\frac{99}{100}\Rightarrow\frac{\sqrt{4\left(\frac{d}{l}\right)^2+1}}{1}=\frac{100}{99}\Rightarrow\left(2\frac{d}{l}\right)^2+1=\left(\frac{1}{1-0.01}\right)^2\Rightarrow\left(2\frac{d}{l}\right)^2+1=(1-0.01)^{-2}$…(4). As per binomial expansion $(1-x)^n=1-nx\vert_{x\ll}$. Accordingly, (4) reduces takes a form $\left(2\frac{d}{l}\right)^2+1=1+0.02\Rightarrow\left(2\frac{d}{l}\right)^2=2\times10^{-2}\Rightarrow\frac{d}{l}=\frac{10^{-1}}{\sqrt{2}}\Rightarrow\frac{d}{l}=0.07$ is the answer.<br>**N.B.:** It involves simple application of algebra. |
| Q-9 | Figure shows a square loop of edge $a$ made of a uniform wire. A current $I$ enters the loop at the point A and leaves it at the point C. Find the magnetic field at the point P which is on the perpendicular bisector of AB at a distance $\frac{a}{2}$ from it.  |
| A-9 | $\frac{2\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{5}}-\frac{1}{3\sqrt{13}}\right)$, coming out of the paper. |
| I-9 | Given geometry is detailed in the figure, on $\hat{\imath}-\hat{\jmath}$ plane. Electrically square loop is made of uniform wire. The current $I$ entering node A routes to exit node C through two paths ABC and ACD of equal lengths $2a$ and hence will have equal resistances. Therefore, the current $I$ will get split into two paths equally at $\frac{I}{2}$.<br>As per Biot-Savart's Laws magnetic field, at a point, due to a wire of length $l$ at a distance $d$ from the midpoint is $B=\frac{\mu_0 I}{2\pi d}\cos\theta$ ...(1). Here, $\theta$ is the angle formed by line joining ends of the wire and the point under consideration, and current carrying wire. Accordingly, magnetic field due to half-length is $B=\frac{\mu_0 I}{4\pi d}\cos\theta$ ...(2). Equation (2) is useful to determine magnetic field due to wire-lengths AD and BC, while equation (1) is useful to determine magnetic field due to wire-lengths DC and AB.<br>In the figure $\cos\theta=\frac{\frac{3}{4}a}{\sqrt{\left(\frac{3}{4}a\right)^2+\left(\frac{a}{2}\right)^2}}\Rightarrow\cos\theta=\frac{3}{\sqrt{13}}$; $\cos\gamma=\frac{\frac{1}{2}a}{\sqrt{\left(\frac{3}{4}a\right)^2+\left(\frac{a}{2}\right)^2}}\Rightarrow\cos\gamma=\frac{2}{\sqrt{13}}$;<br>$\cos\alpha=\frac{\frac{1}{4}a}{\sqrt{\left(\frac{1}{4}a\right)^2+\left(\frac{a}{2}\right)^2}}\Rightarrow\cos\alpha=\frac{1}{\sqrt{5}}$; $\cos\beta=\frac{\frac{1}{2}a}{\sqrt{\left(\frac{1}{4}a\right)^2+\left(\frac{a}{2}\right)^2}}\Rightarrow\cos\beta=\frac{2}{\sqrt{5}}$<br>Direction of magnetic field at point P is determined using Ampere's Right-Hand Thumb Rule; as per direction convention magnetic field due to wire ABC, coming out of paper, is assigned $(\hat{k})$ direction and magnetic field due to wire ADC, entering into the plane of paper is assigned direction $(-\hat{k})$.<br>Accordingly, magnetic field at point P due to each branch of wire, using the available data, is as under –<br>**Wire AB:** Using (1); $\vec{B}_{AB}=\frac{\mu_0\frac{I}{2}}{2\pi\frac{a}{4}}\left(\frac{2}{\sqrt{5}}\right)(\hat{k})\Rightarrow\vec{B}_{AB}=\frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k})$<br>**Wire BC:** Using (2) for sections BF and FC; $\vec{B}_{BC}=\vec{B}_{BF}+\vec{B}_{FC}\Rightarrow\vec{B}_{BC}=\frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{1}{\sqrt{5}}\right)(\hat{k})+\frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{3}{\sqrt{13}}\right)(\hat{k})$. It leads to $\vec{B}_{BC}=\frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}}+\frac{3}{\sqrt{13}}\right](\hat{k})$<br>**Wire AD:** Using (2) for sections AE and ED; $\vec{B}_{AD}=\vec{B}_{AE}+\vec{B}_{ED}\Rightarrow\vec{B}_{AD}=\frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{1}{\sqrt{5}}\right)(-\hat{k})+\frac{\mu_0\frac{I}{2}}{4\pi\frac{a}{2}}\left(\frac{3}{\sqrt{13}}\right)(-\hat{k})$. It leads to $\vec{B}_{AD}=\frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}}+\frac{3}{\sqrt{13}}\right](-\hat{k})$ |

| | |
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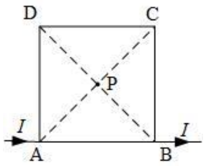
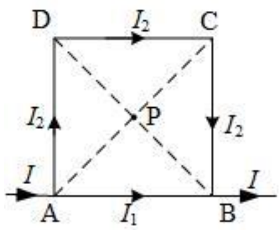
| | **Wire DC:** Using (1); $\vec{B}_{DC} = \frac{\mu_0 \frac{I}{2}}{2\pi \frac{3a}{4}}\left(\frac{2}{\sqrt{13}}\right)(-\hat{k}) \Rightarrow \vec{B}_{DC} = \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k})$.<br>Thus, resultant magnetic field at P is $\vec{B} = \vec{B}_{AB} + \vec{B}_{BC} + \vec{B}_{AD} + \vec{B}_{DC} = \frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k}) + \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](\hat{k}) + \frac{\mu_0 I}{4\pi a}\left[\frac{1}{\sqrt{5}} + \frac{3}{\sqrt{13}}\right](-\hat{k}) + \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k}) \Rightarrow \vec{B} = \frac{2\mu_0 I}{\sqrt{5}\pi a}(\hat{k}) + \frac{2\mu_0 I}{3\sqrt{13}\pi a}(-\hat{k})$.<br>It leads to $\vec{B} = \frac{2\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right)\hat{k}$, **or** $\boldsymbol{B} = \frac{2\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right)$ **coming out of the paper.**<br>**N.B.:** Solution of this problem could be simplified using geometrical symmetry of sides AD and BC, as well as asymmetry of sides DC and AB w.r.t. point P. Yet, for clarity of concepts the problem. |
| Q-10 | Figure shows a square loop of edge $a$ made of a uniform wire. A current $i$ enters the loop at the point A and leaves it at the point B. Find the magnetic field at the center of the loop.<br><br> |
| A-10 | Zero |
| I-10 | Given system is shown in figure, on $\hat{\imath} - \hat{\jmath}$ plane. The square loop is made of uniform wire and therefore resistances $R_1$ and $R_2$ of the path AB and ADC would be proportional to their lengths, accordingly, $\frac{R_1}{R_2} = \frac{1}{3}$. Further, both the paths are parallel between AB and currents in the two paths shall be $\frac{I_1}{I_2} = \frac{3}{1}$, such that $I_1 + I_2 = I$. Applying componendo to the ratio of currents $\frac{I}{I_2} = 4 \Rightarrow I_2 = \frac{I}{4}$, and $I_1 = \frac{3I}{4}$.<br><br><br>Geometrically, distance of point P, being midpoint, from current carrying branches AB, BC, AD and DC, each of length $a$, is $d = \frac{a}{2}$.<br>As per Biot-Savart's Laws magnitude of magnetic field, at a point, due to a wire of length $l$ at a distance $d$ from the midpoint is $B = \frac{\mu_0 i}{2\pi d}\cos\theta$. Here, $i$ is the current through the wire $\theta$ is the angle formed by line joining ends of the wire and the point under consideration, and current carrying wire. Geometrically, $\theta = 45^0 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$. Accordingly, magnitude of magnetic field due to half-length is $B = \frac{\mu_0 i}{2\sqrt{2}\pi d}$ ...(1).<br>As per Ampere's Right-Hand Thumb Rule magnetic field at point P due to wire AD, DC and CB carrying equal current $I_2 = \frac{I}{4}$ would be equal and entering plane of the figure i.e. along $(-\hat{k})$. Therefore, magnetic field at P due to these three branches would be $\vec{B}_2 = 3\left(\frac{\mu_0 \frac{I}{4}}{2\sqrt{2}\pi d}\right)(-\hat{k}) \Rightarrow \vec{B}_2 = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(-\hat{k})$. Likewise, due to wire AB carrying current $I_2 = \frac{3I}{4}$ would be leaving plane of the figure i.e. along $(\hat{k})$. Thus magnetic field due to AB would be $\vec{B}_1 = \frac{\mu_0 \frac{3I}{4}}{2\sqrt{2}\pi d}(\hat{k}) \Rightarrow \vec{B}_1 = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(\hat{k})$.<br>Thus net magnetic field at P is $\vec{B} = \vec{B}_1 + \vec{B}_2 \Rightarrow \vec{B} = \frac{3\mu_0 I}{8\sqrt{2}\pi d}(\hat{k}) + \frac{3\mu_0 I}{8\sqrt{2}\pi d}(-\hat{k}) \Rightarrow \boldsymbol{B} = \boldsymbol{0}$ **is the answer**.<br>**N.B.:** This can be made an objective problem also. |

**Important Note:** You may encounter need of clarification on contents and analysis or an inadvertent typographical error. We would gratefully welcome your prompt feedback on mail ID: subhashjoshi2107@gmail.com. If not inconvenient, please identify yourself to help us reciprocate you suitably.