Electromagnetism: Current Electricity – Selected Questions (Part-3)

Magnetism and Magnetic Properties

Important Note

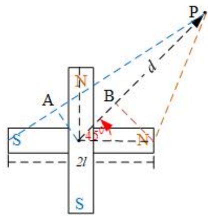
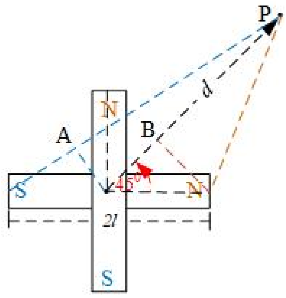
1. Magnetism in an important and inseparable part of electromagnetism. Ability to apply concepts of magnetism is helpful in gaining proficiency in understanding electromagnetism and its applications.

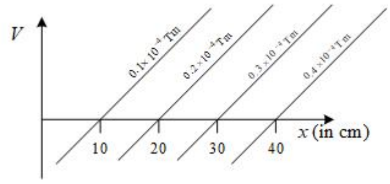
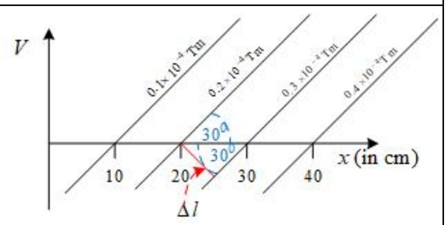
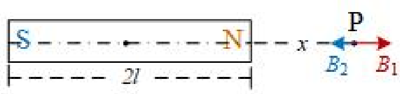
2. A student at a stage to refer to these questions and illustrations is expected to have attained a reasonable understanding of concepts and visualization. Moreover, forward journey involves integration of concepts on a wider canvas. Therefore, illustrations have been made a bit crisp. This would help students to harness their understanding at a faster rate.

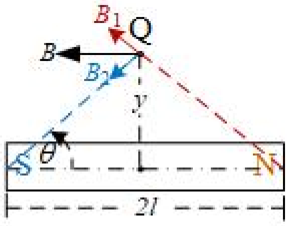
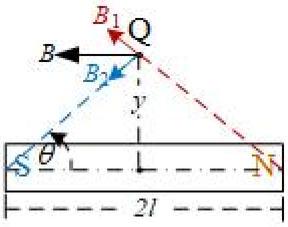
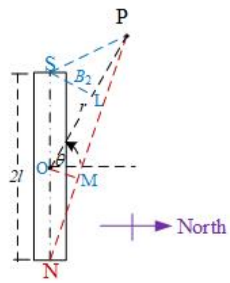
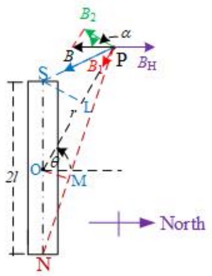
3. Avoid fatigue due to long and continuous sitting in solving such problems. Take a reasonable break to refresh before taking next part. Gradually, capability to withstand fatigue will grow to enable you to take up bigger challenges.

4. Electromagnetism is a subject so closely intertwined that discretization of problems on Magnetism and Magnetic Effect of Electric Current fails as one goes ahead. This is brought out in footnote of illustration of such problems.

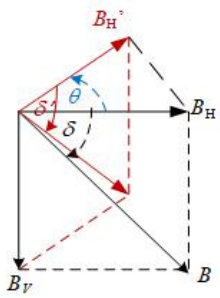
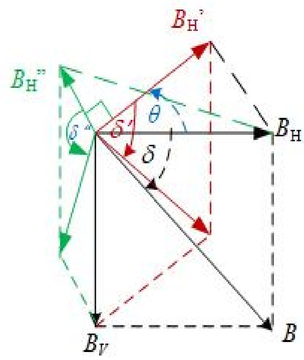
Q-1	Magnetic scalar potential is defined as $U(\vec{r}_2)-U(\vec{r}_1)=\int_{\vec{r}_1}^{\vec{r}_2}\vec{B}.d\vec{l}$. Apply this equation to a closed curve enclosing a long straight wire. The RHS of the above equation is then $(-)\mu_0 i$ by Ampere's Law. We see that $U(\vec{r}_2)-U(\vec{r}_1)$ even when $\vec{r}_2=\vec{r}_1$.
A-1	No
I-1	As per definition magnetic scalar potential combined is $U(\vec{r}_2)-U(\vec{r}_1)=\int_{\vec{r}_1}^{\vec{r}_2}\vec{B}.d\vec{l}$. Whereas, as per Ampere's law, line-integral of magnetic field around a conductor carrying current i is $\int_{\vec{r}_1}^{\vec{r}_2}\vec{B}.d\vec{l}=(-)\mu_0 i$. Here, $\vec{B}$ is the magnetic field intensity. Whereas, $\vec{B}$ due to a magnetic pole of pole strength m is $\vec{B}=\frac{\mu_0}{4\pi}\times\frac{m}{r^2}\hat{r}$. Accordingly, scalar magnetic potential $U(r)=-\int_{\infty}^{r}\vec{B}.d\vec{r}=-\int_{\infty}^{r}\left(\frac{\mu_0}{4\pi}\times\frac{m}{r^2}\hat{r}\right).d\vec{r}\Rightarrow U(r)=-\frac{\mu_0 m}{4\pi}\int_{\infty}^{r}\frac{1}{r^2}.dr=-\frac{\mu_0 m}{4\pi}\left[-\frac{1}{r}\right]_{\infty}^{r}\Rightarrow U(r)=\frac{\mu_0 m}{4\pi r}$. It is seen that both the definitions the definitions are conceptually different, in earlier the cause is electric current while in the latter the cause is a static magnetic pole. Thus both the equations cannot be correlated at this stage, hence, **answer is No.**
Q-2	The reduction factor K of a tangent galvanometer is written on the instrument. The manual says that the current is obtained by multiplying this factor to $\tan\theta$. The procedure works well at Bhubaneshwar. Will the procedure work if the instrument is taken to Nepal? If there is some error, can it be corrected by correlating the manual or the instrument will have to be taken back to the factory?
A-2	Yes, No
I-2	In tangent galvanometer, at a place $\tan\theta=\frac{B}{B_H}$, here B_H is the horizontal component of the earth's magnetic field B_E, and $B=\frac{\mu_0 ni}{2r}$ is the magnetic field producied by current i in the circular coil of radius r and number of turns n. The $B_H=B_E\cos\delta$, here δ is the angle of dip and both B_E and δ varies along the meridian as one mover from equator to magnetic poles of the earth and so also B_H is place specific.

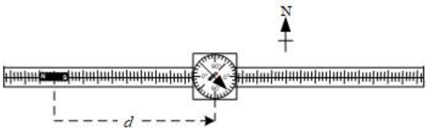
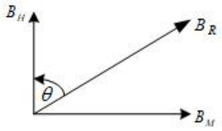
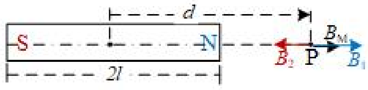
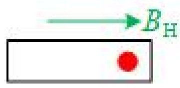
	At equilibrium position at deflection θ of the needle of galvanometer $B\sin\theta = B_H\cos\theta \Rightarrow \tan\theta = \frac{B}{B_H}$. It leads to $\tan\theta = \frac{\frac{\mu_0 n i}{2r}}{B_H} \Rightarrow \tan\theta = \frac{\mu_0 n}{2rB_H} i \Rightarrow i = \frac{2rB_H}{\mu_0 n}\tan\theta \Rightarrow i = K\tan\theta$. Since, numerically $K < 1$ and hence it is called reduction factor. Here, $K \propto B_H$, and hence the procedure will work at Nepal and error can be corrected by correlating $K' = K\frac{B_H'}{B_H}$, where B_H' corresponds to Nepal while B_H corresponds to Bhubaneshwar. Thus, **answer to first part is Yes,** while answer to **second part** on need of taking instrument to factory **is No.**
Q-3	Two short magnets of equal dipole moments M are fastened at the center as shown in the figure. Magnitude of the magnetic field at a distance d from the center on the bisector of the right angle is  (a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$ (c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$
A-3	(c)
I-3	Given that Two dipoles of pole strength m and magnetic moment $M = 2ml$ are short in length of magnets $2l \ll d$. The magnets are fastened at their centers as shown in the figure (not to the scale). Therefore, magnetic field at the point P due to North poles of one of the two magnets, at a distance $r_N \approx BP = d - l\cos 45^0 \Rightarrow r_n = d - \frac{l}{\sqrt{2}}$ would be $B_N = \frac{\mu_0}{4\pi}\frac{m}{\left(d-\frac{l}{\sqrt{2}}\right)^2}$. Likewise, due to south pole at a distance $r_S = d + \frac{l}{\sqrt{2}}$ it would be $B_S = \frac{\mu_0}{4\pi}\frac{(-)m}{\left(d+\frac{l}{\sqrt{2}}\right)^2}$. Thus, net magnetic field due to dipole is  $B_1 = B_N + B_S$. It leads to $B_1 = \frac{\mu_0}{4\pi}\frac{m}{\left(d-\frac{l}{\sqrt{2}}\right)^2} - \frac{\mu_0}{4\pi}\frac{m}{\left(d+\frac{l}{\sqrt{2}}\right)^2} = \frac{\mu_0 m}{4\pi}\left(\frac{1}{\left(d-\frac{l}{\sqrt{2}}\right)^2} - \frac{1}{\left(d+\frac{l}{\sqrt{2}}\right)^2}\right)$. It further, solves into $B_1 = \frac{\mu_0 m}{4\pi}\left(\frac{\left(d+\frac{l}{\sqrt{2}}\right)^2-\left(d-\frac{l}{\sqrt{2}}\right)^2}{\left(d^2-\frac{l^2}{2}\right)^2}\right) \approx \frac{\mu_0 m}{4\pi}\left(\frac{\left(2\times\frac{l}{\sqrt{2}}\right)\times 2d}{d^4}\right) \Rightarrow B_1 = \frac{\mu_0(m\times 2l)}{4\pi}\left(\frac{\sqrt{2}}{d^3}\right) \Rightarrow B_1 = \frac{\mu_0 M}{4\pi}\left(\frac{\sqrt{2}}{d^3}\right)$. Likewise, magnetic field due to another magnet is $B_2 = \frac{\mu_0 M}{4\pi}\left(\frac{\sqrt{2}}{d^3}\right)$, both are of same magnitude and identical signs and hence, net magnetic field at point P is $B = B_1 + B_2 = 2B_1 = 2\times\frac{\mu_0 M}{4\pi}\left(\frac{\sqrt{2}}{d^3}\right) \Rightarrow B = \frac{\mu_0 M}{4\pi}\left(\frac{2\sqrt{2}}{d^3}\right)$. This expression matches with that in the **option (c), is the answer.**
Q-4	A tangent galvanometer is connected directly to an ideal battery. If the number of turns in the coil is doubled the deflection will (a) Increase (b) Decrease (c) Remain unchanged (d) Either increase or decrease
A-4	(c)
I-4	In tangent galvanometer, at a place $\tan\theta = \frac{B}{B_H}$, here B_H is the horizontal component of the earth's magnetic field B_E, and $B = \frac{\mu_0 n i}{2r}$ is the magnetic field producied by current i in the circular coil of radius r and number of turns n. The $B_H = B_E\cos\delta$, here δ is the angle of dip and both B_E and δ varies along the meridian as one mover from equator to magnetic poles of the earth and so also B_H is place specific. At equilibrium position at deflection θ of the needle of galvanometer $B\sin\theta = B_H\cos\theta \Rightarrow \tan\theta = \frac{B}{B_H}$. It leads to $\tan\theta = \frac{\frac{\mu_0 n i}{2r}}{B_H} \Rightarrow \tan\theta = \frac{\mu_0 n}{2rB_H} i \Rightarrow i = \frac{2rB_H}{\mu_0 n}\tan\theta \Rightarrow i = K\tan\theta$….(1). Here, reduction factor $K = \frac{2rB_H}{\mu_0 n}$ …(2)

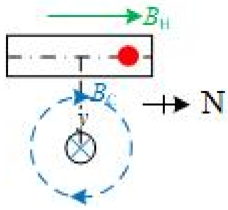
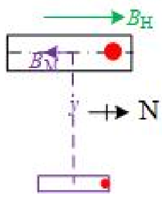
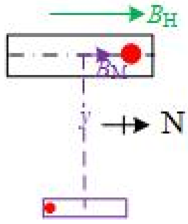
	The question states that tangent galvanometer is directly connected to an ideal battery of emf E. It implies that the only resistance in the circuit is the resistance of the turns which is $R=\rho\frac{L}{A}$, here restivity of material of turns ρ and area of cross-section A of the wire forming turns remain same and length of wire is $L=2\pi rn$ where r is the radius of the turns and n is the number of turns. Therefore, when number of turns are doubled i.e. $n'=2n$, length of turns would be $L'=2\pi rn' \Rightarrow L'=4\pi rn$. Current through the toil of the galvanometer in this case as per Ohm's Law is initially $i=\frac{E}{R}=\frac{E}{\rho\frac{2\pi rn}{A}} \Rightarrow i=\frac{EA}{\rho\times 2\pi rn} \Rightarrow i=\frac{EA}{2\rho\pi rn}$. Accordingly, deflection as per (1) would be $\frac{EA}{2\rho\pi rn}=\frac{2rB_H}{\mu_0 n}\tan\theta$. Thus def;ection would be $\tan\theta=\frac{EA}{2\rho\pi rn}\times\frac{\mu_0 n}{2rB_H} \Rightarrow \theta=\tan^{-1}\left(\frac{\mu_0 EA}{4\pi\rho r^2 B_H}\right)$…(2). It is seen that deflection θ is not a function of number of turns n. Hence, in the given case deflection will not change with increase in number of turns. This conclusion is in accordance with **option (c), is the answer.**
Q-5	Figure shows some of the equipotential surfaces of the magnetic scalar potential. Find the magnetic field B at a point in the region. All equipotential lines are inclined to X-axis at $\theta=30^0$ 
A-5	2.0×10^{-4} T
I-5	We know that scalar magnetic potential difference $\Delta V=\vec{B}.\Delta\vec{l} \Rightarrow \Delta V=B\Delta l\cos\alpha \Rightarrow B=\frac{\Delta V}{\Delta l\cos\alpha}$…(1). Here, $\Delta l=\Delta x\cos\alpha$ is separation between the equipotential lines. It is seen that potential difference between the equipotential lines is $\Delta V=0.1\times10^{-4}$Tm, intercepts of the equipotential lines on X-axis are at $\Delta V=0.10$ m, and angle $\alpha=90^0-\theta=90^0-30^0 \Rightarrow \alpha=60^0$. Using the given data in (1) $B=\frac{0.1\times10^{-4}}{0.10\times\cos 60^0}=\frac{0.1\times10^{-4}}{0.05} \Rightarrow B=\mathbf{2.0\times10^{-4}}$ **is the answer.** 
Q-6	The magnetic field at a point, 10 cm away from a magnetic pole, is found to be 2.0×10^{-4} T. Find the magnetic moment of the dipole if the point is (a) In the end-on position of the dipole (b) In Broadside-on position of the pole
A-6	(a) 1.0 Am^2, and (b) 2.0 Am^2
I-6	The problem has two parts and each is solved separately with figure (not to proportion). **Part (a):** It is given that in End-on position magnetic field at a point P at a distance $x=0.10$m is $B=2.0\times10^{-4}$ T and it required to find magnetic moment of the dipole $M=m\times 2l=2ml$.  Magnetic field at P due to north pole of polarity m is $\vec{B}_1=\frac{\mu_0}{4\pi}\times\frac{m}{x^2}\hat{x}$ and due to south pole of polarity $(-m)$ is $\vec{B}_2=\frac{\mu_0}{4\pi}\times\frac{(-m)}{(x+2l)^2}\hat{x} \Rightarrow \vec{B}_2=-\frac{\mu_0}{4\pi}\times\frac{m}{(x+2l)^2}\hat{x}$. Therefore, net-magnetic field at the point is $\vec{B}=\frac{\mu_0 m}{4\pi}\left(\frac{1}{x^2}-\frac{1}{(x+2l)^2}\right)\hat{x}=\frac{\mu_0 m}{4\pi}\left(\frac{(x+2l)^2-x^2}{x^2(x+2l)^2}\right)\hat{x}$. In case of dipole $x\gg 2l$ and hence $(2l)^2\to 0$ and $x+2l\to x$ Therefore, magnitude of magnetic field it approximates to $B=\frac{\mu_0}{4\pi}\times m\left(\frac{4lx}{x^2x^2}\right) \Rightarrow B=\frac{\mu_0}{4\pi}\times\frac{2(2ml)}{x^3} \Rightarrow B=\frac{\mu_0}{4\pi}\times\frac{2M}{x^3} \Rightarrow M=\frac{4\pi}{\mu_0}\times\frac{Bx^3}{2x^3}$. Using the available data, $M=\frac{1}{10^{-7}}\times\frac{2.0\times10^{-4}\times(0.1)^3}{2}=$ **1 A-m^2, is the answer.**

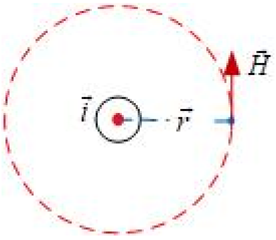
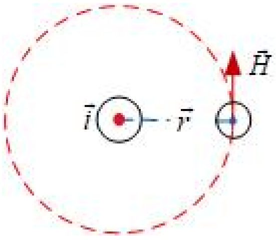
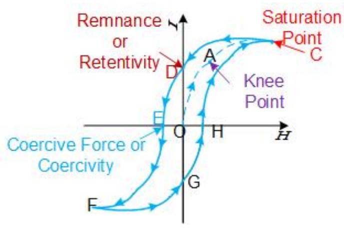
	Part (b): Except the geometry, as shown in the figure, logic is same as in part (s), In this case $\vec{B}_1$ and $\vec{B}_2$ are of equal magnitudes but in directions as shown in the figure. And resultant magnetic field would be $B = 2B_1\cos\theta$.  In the figure $\cos\theta = \frac{l}{\sqrt{y^2+l^2}}$ and $B_1 = \frac{\mu_0}{4\pi}\times\frac{m}{y^2+l^2} \Rightarrow B = \frac{\mu_0}{4\pi}\times\frac{2ml}{(y^2+l^2)^{\frac{3}{2}}}$. This is approximated to $B = \frac{\mu_0}{4\pi}\times\frac{M}{y^3} \Rightarrow M = By^3\times\frac{4\pi}{\mu_0}$. Using the given data $M = 2.0\times10^{-4}\times(0.1)^3\times\frac{1}{10^{-7}} = $ **2.0 A-m², is the answer.**
Q-7	A bar magnet has a length of 8 cm. The magnetic field at a point at a distance 3 cm from the center in the broadside-on position is found to be 4×10^{-6} T. Find the pole strength of the magnet.
A-7	6×10^{-2}Am
I-7	 Except the geometry, as shown in the figure, logic is same as in part (s), In this case $\vec{B}_1$ and $\vec{B}_2$ are of equal magnitudes but in directions as shown in the figure. And resultant magnetic field would be $B = 2B_1\cos\theta$. The length of the bar magnet $2l = 0.08 \Rightarrow l = 0.04$ m and point on broadside-on position is at $y = 0.03$ m. As shown in the figure $\cos\theta = \frac{l}{\sqrt{y^2+l^2}}$ and $B_1 = \frac{\mu_0}{4\pi}\times\frac{m}{y^2+l^2} \Rightarrow B = \frac{2\mu_0 m\cos\theta}{4\pi} \Rightarrow$ $B = \frac{2\mu_0 m}{4\pi(y^2+l^2)}\times\frac{l}{\sqrt{y^2+l^2}} = \frac{\mu_0}{4\pi}\times\frac{2ml}{4\pi(y^2+l^2)^{\frac{3}{2}}}$. Using the available data $4\times10^{-6} = 10^{-7}\times\frac{m\times0.08}{(0.03^2+0.04^2)^{\frac{3}{2}}} \Rightarrow 4 = \frac{0.08\times10^{-1}m}{(0.05)^3} \Rightarrow m = \frac{125\times10^{-6}}{2\times10^{-3}} = 62.5\times10^{-3}$ Am. Using principle of SDs $m =$ **6×10^{-2}Am is the answer.** **N.B.:** This is a good case of application of principle of SDs.
Q-8	A magnetic dipole of the magnetic moment $0.72\sqrt{2}$ A.m² is placed horizontally with the north pole pointing towards east. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.
A-8	20 cm from the dipole, at an angle $\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ West of North is the answer..
I-8	Magnetic dipole having magnetic moment $M = 2ml = 0.72\sqrt{2}$ A.m² is placed horizontally. It is given that horizontal component of earth's magnetic field is $B_H = 18\times10^{-6}$ T.  In the problem statement north pole of magnet is oriented towards East as shown in the figure. This case needs to be handled differently from Broadside-On and End-on positions, and for this magnetic potential at a point defined with radial coordinates (r,θ) have been chosen, as shown in the figure, instead of Cartesian coordinates.  Given that a dipole of pole strength m and magnetic moment $M = 2ml$ are short in length of magnets $2l \ll r$. The magnets are fastened at their centers as shown in the figure (not to the scale). Therefore, magnetic field at the point P due to North poles of one of the two magnets, at a distance $r_N \approx BP = r + l\sin\theta$ would be $V_N = \frac{\mu_0}{4\pi}\frac{m}{(r+l\sin\theta)}$. Likewise, due to south pole at a distance $r_S = r - l\sin\theta$ it would be $V_S = \frac{\mu_0}{4\pi}\frac{(-m)}{(r-l\sin\theta)}$. Thus, net magnetic field due to dipole is $V = V_N + V_N$. It leads to $V = \frac{\mu_0}{4\pi}\frac{m}{(r+l\sin\theta)} - \frac{\mu_0}{4\pi}\frac{m}{(r-l\sin\theta)} = \frac{\mu_0 m}{4\pi}\left(\frac{(r-l\sin\theta)-(r-l\sin\theta)}{(r^2-l^2\sin^2\theta)}\right) \Rightarrow V = -\frac{\mu_0 m}{4\pi}\left(\frac{2l\sin\theta}{(r^2-l^2\sin^2\theta)}\right)$. Since, $r^2 \gg (l\sin\theta)^2$. Therefore, $V = -\frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{r}\right)$.

	Magnetic potential $[V = f(\theta, r)]$is a scalar quantity whereas magnetic field intensity is a vector quantity $B = -\frac{\partial V}{\partial r}$, such that its radial component is $B_1 = \frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(-\frac{\mu_0}{4\pi}\left(\frac{M\sin\theta}{d^2}\right)\right) = \frac{\mu_0 M\sin\theta}{4\pi} \times \frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) \Rightarrow B_1 = \frac{\mu_0 M\sin\theta}{4\pi} \times \left(-\frac{2}{r^3}\right)$. It leads to $B_1 = -\frac{2\mu_0 M\sin\theta}{4\pi r^3}$. Likewise, tangential component of magnetic field is $B_2 = -\frac{\partial V}{\partial(r\theta)}$, here infinitesimal tangential displacement is $\partial(r\theta) = r\,(\partial\theta)$ is the tangential displacement. Accordingly, $B_2 = -\frac{1}{r} \times \frac{\partial}{\partial\theta}\left(\frac{\mu_0 M}{4\pi r^2} \times \sin\theta\right) = -\frac{\mu_0 M}{4\pi r^3} \times \frac{\partial}{\partial\theta}\sin\theta$. It solves into, $B_2 = -\frac{\mu_0 M}{4\pi r^3} \times \cos\theta$. Geometrically, as shown in the figure $\tan\alpha = \frac{B_1}{B_2} = \frac{-\frac{2\mu_0 M\sin\theta}{4\pi r^3}}{-\frac{\mu_0 M}{4\pi r^3}\times\cos\theta}$. It leads to $\tan\alpha = 2\tan\theta$. As seen from the diagram, net magnetic field B due to dipole be equal and opposite to B_H makes an angle $\alpha = 90^0 - \theta$ with the magnetic axis. Therefore, $\tan\alpha = \tan(90^0 - \theta) = \cot\theta = 2\tan\theta \Rightarrow \tan^2\theta = \frac{1}{2}$. It leads to that $\tan\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$. Further, magnitude of magnetic field due to dipole is $B = \sqrt{\left(\frac{2\mu_0 M\sin\theta}{4\pi r^3}\right)^2 + \left(\frac{\mu_0 M}{4\pi r^3} \times \cos\theta\right)^2}$. It, further solves into $B = \frac{\mu_0}{4\pi} \times \frac{M\cos\theta}{r^3}\sqrt{\left(2 \times \frac{\sin\theta}{\cos\theta}\right)^2 + 1} \Rightarrow B = \frac{\mu_0}{4\pi} \times \frac{M\cos\theta}{r^3}\sqrt{(2 \times \tan\theta)^2 + 1} \Rightarrow B = \frac{\mu_0}{4\pi} \times \frac{M\cos\theta}{r^3}\sqrt{\left(\sqrt{2}\right)^2 + 1}$. Further, $\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+\frac{1}{2}}} \Rightarrow \cos\theta = \sqrt{\frac{2}{3}}$. Accordingly, $B = \frac{\mu_0}{4\pi} \times \frac{M}{r^3} \times \sqrt{\frac{2}{3}} \times \sqrt{3} \Rightarrow B = \frac{\mu_0}{4\pi} \times \frac{M}{r^3} \times \sqrt{2}$. Requirement of Null point is $B = B_H$. Therefore, using the available data $10^{-7} \times \frac{0.72\sqrt{2}}{r^3} \times \sqrt{2} = 18 \times 10^{-6} \Rightarrow r = \sqrt[3]{\frac{1.44\times10^{-1}}{18}} = 0.2$m or 20 cm from the center of the dipole. Thus, the null point is at 20 cm from the center of the pole with South pole at an angle $\theta = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ West of North is the answer.. **N.B.: 1.** The question could be solved by determining combined magnetic field at a point due to North and south Pole and then position of its equilibrium with B_H to ascertain the null point. Instead, scalar magnetic potential at a point has been determined and its radial and tangential components have been determined to arrive at the position of null point. This makes analysis and solution simple. **2.** Use of standard formulae apparently makes solution simple and fast. But, in typical problems like this ability to evolve solution from first principle builds confidence to handle any unknown problem, generally encountered in real life. This is the basic purpose of education.
Q-9	A magnetic needle is free to rotate in a vertical plane which makes an angle of 60^0 with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ with the horizontal, what would be the dip at that place?
A-9	30^0

I-9	 Geo-magnetic field in magnetic meridian has been conceptualized with Dip Circle as show in the figure. Here, $B_H = B\cos\delta$ is horizontal component and $B_V = B\sin\delta$ is vertical component of earth's magnetic field B. Accordingly, $\tan\delta = \frac{B_V}{B_H} \Rightarrow B_V = B_H \tan\delta$…(1). In a vertical plane at an angle $\theta = 60^0$ magnetic needle stay at a new dip angle $\delta' = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$. Thus, horizontal component of magnetic field in the vertical plane $B'_H = B_H\cos\theta$ …(2). Accordingly, it gives new dip angle δ' such that $\tan\delta' = \frac{B_V}{B_H{}'} \Rightarrow B_V = B_H{}'\tan\delta'$…(3). Combining (1), (2) and (3) we get $B_H\tan\delta = (B_H\cos\theta)\times\tan\delta'$. It leads to $\tan\delta = \cos\theta\times\tan\delta'$. Using the given data $\tan\delta = \cos 60^0 \times \tan\left(\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)\right) \Rightarrow \tan\delta = \frac{1}{2}\times\frac{2}{\sqrt{3}} \Rightarrow \tan\delta = \frac{1}{\sqrt{3}} = \tan 30^0$. It implies $\delta = 30^0$**, is the answer.** **N.B.:** A 3-D figure helps to visualize and analyze problems and has been used in illustration to make it self-explanatory.
Q-10	The needle of a dip circle shows an apparent dip of 45^0 in a particular position and 53^0 when the circle is rotated through 90^0. Find the true dip.
A-10	39^0
I-10	Angle of dip in magnetic meridian is called here as true angle of dip δ, and it is to be determined. The problem is being analyzed in context of Dip-Circle. In magnetic meridian Here, $B_H = B\cos\delta$ …(1) is horizontal component and $B_V = B\sin\delta$…(2) is vertical component of earth's magnetic field B. Accordingly, $\tan\delta = \frac{B_V}{B_H} \Rightarrow B_V = B_H\tan\delta$…(3). Here, δ is true dip angle. In a particular position on a plane at an angle θ the dip angle is measured $\delta' = 45^0$. Thus, horizontal component of magnetic field in the vertical plane $B'_H = B_H\cos\theta$ …(4). Accordingly, it gives new dip angle δ' such that $\tan\delta' = \frac{B_V}{B_H{}'} \Rightarrow B_V = B_H{}'\tan\delta'$…(5). Combining (4) and (5) $B_V = (B_H\cos\theta)\tan\delta'$…(6). In another plane when dip circle is rotated through 90^0 the horizontal component of geo-magnetic field is $B''_H = B_H\cos(90^0+\theta) \Rightarrow B''_H = B_H\sin\theta$…(7). Therefore, $\tan\delta'' = \frac{B_V}{B''_H} \Rightarrow \tan\delta'' = \frac{B_V}{B_H\sin\theta} \Rightarrow B_V = (B_H\sin\theta)\times\tan\delta''$…(8). Combining (6) and (7), we have $(B_H\cos\theta)\tan\delta' = (B_H\sin\theta)\times\tan\delta''$. It leads to $\sin\theta\tan\delta'' = \cos\theta\tan\delta' \Rightarrow \tan\theta = \frac{\tan\delta'}{\tan\delta''}$. Using the given data where $\delta' = 45^0$ and $\delta'' = 53^0$ we have $\tan\theta = \frac{\tan 45^0}{\tan 53^0} \Rightarrow \tan\theta = \cot 53^0 = 0.7536$…(9). In another plane when dip circle is rotated through 90^0 the horizontal component of geo-magnetic field is $B''_H = B_H\cos(90^0+\theta) \Rightarrow B''_H = B_H\sin\theta$…(7). Therefore, $\tan\delta'' = \frac{B_V}{B''_H} \Rightarrow \tan\delta'' = \frac{B_V}{B_H\sin\theta} \Rightarrow B_V = (B_H\sin\theta)\times\tan\delta''$…(8).  Combining (6) and (7), we have $(B_H\cos\theta)\tan\delta' = (B_H\sin\theta)\times\tan\delta''$. It leads to $\sin\theta\tan\delta'' = \cos\theta\tan\delta' \Rightarrow \tan\theta = \frac{\tan\delta'}{\tan\delta''}$. Using the given data where $\delta' = 45^0$ and $\delta'' = 53^0$ we have $\tan\theta = \frac{\tan 45^0}{\tan 53^0} \Rightarrow \tan\theta = \cot 53^0 = 0.7536$…(9). Combining (3) and (6), $B_H\tan\delta = (B_H\cos\theta)\tan\delta' \Rightarrow \tan\delta = \cos\theta\tan\delta'$…(10).

	Trigonometrically $\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+(0.7536)^2}} \Rightarrow \cos\theta = 0.7986$, using it in (10) $\tan\delta = 0.7986 \times 1$ we have $\tan\delta = 0.7986 \Rightarrow \delta = 38^0 37'$ say **39^0 is the answer.** **N.B.:** A 3-D figure helps to visualize and analyze problems and has been used in illustration to make it self-explanatory.
Q-11	A short magnet produces a deflection of 37^0 in a deflection magnetometer in Tan-A position when placed at a separation of 10 cm from the needle. Find the ratio of the magnetic moment of the magnet to the earth's horizontal magnetic field.
A-11	3.75×10^3 Am^2/T
I-11	Given is short magnet having magnetic moment $M = 2ml$ placed in Tan-A position of the deflection magnetometer as shown in the figure. It is equivalent to end-on position. The needle is deflected by an angle $\theta = 37^0$.  Magnetic needle is subjected to two magnetic fields B_M produced by the short magnet and B_H the horizontal component of geo-magnetic field. Accordingly, $\tan\theta = \frac{B_M}{B_H}$ ...(1), as shown in the figure. Since, deflection magnetometer is placed horizontally and hence in analysis of the problem only $B_H$is relevant.  Magnetic field at P due to north pole of polarity m is $\vec{B}_1 = \frac{\mu_0}{4\pi} \times \frac{m}{(d-l)^2}\hat{x}$ and due to south pole of polarity $(-m)$ is $\vec{B}_2 = -\frac{\mu_0}{4\pi} \times \frac{(m)}{x^2}\hat{x} \Rightarrow \vec{B}_2 = -\frac{\mu_0}{4\pi} \times \frac{m}{(d+l)^2}\hat{x}$.  Therefore, net-magnetic field at the point is $\vec{B}_M = \frac{\mu_0 m}{4\pi}\left(\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right)\hat{x} \Rightarrow \vec{B}_M = \frac{\mu_0 m}{4\pi}\left(\frac{(d+l)^2-(d-l)^2}{(d^2-l^2)^2}\right)\hat{x} \Rightarrow B_M = \frac{\mu_0 m}{4\pi} \times \frac{4dl}{(d^2-l^2)^2}$. In case of dipole $d \gg 2l$ and hence $(d^2-l^2)^2 \to d^4$, therefore, $B_M = \frac{\mu_0}{4\pi} \times \frac{4mld}{d^4} \Rightarrow B_M = \frac{\mu_0}{4\pi} \times \frac{2M}{d^3}$...(2). Combining (1) and (2), $\tan\theta = \frac{\frac{\mu_0}{4\pi}\times\frac{2M}{d^3}}{B_H} \Rightarrow \frac{M}{B_H} = \frac{4\pi}{\mu_0} \times \frac{(0.1)^3}{2} \times \tan\theta = \frac{(0.1)^3}{2\times10^{-7}} \times \tan 37^0 = (0.5\times10^4) \times 0.75 \Rightarrow \frac{M}{B_H} = $ **$3.75 \times 10^3 \text{Am}^2/\text{T}$ is the answer.** **N.B.:** Decomposing the problem in stages with necessary diagrams adds clarity, ability to solve unknown problems and burden of remembering formulae.
Q-12	A short magnet oscillates in an oscillation magnetometer with a time period of 0.10 s where the earth's horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire placed 20 cm east of the magnet. Find the new time period.
A-12	0.076 s
I-12	This question magnetic effect of current as per Biot-Sevart's Law to determine magnetic field due to current carrying conductor, in addition to the concepts of magnetism. In an oscillation magnetometer $M = \left(\frac{2\pi}{T}\right)^2 \times \frac{I}{B_H}$...(1). Here, M is magnetic moment of the suspended magnet, I is the moment of inertia of the magnet, B_H is the horizontal component of the magnetic field where the instrument is placed and T is the time period of the oscillation. The (1) can be expressed as $T = 2\pi\sqrt{\frac{I}{MB_H}}$...(2). It is given that time period $T = 0.10$ s with barely the magnet is used in the instrument.  Next a vertical wire, placed $y = 0.20$ m east of the magnet, carries a downward current $I = 18$A. Thus current modifies magnetic field affecting oscillation of the magnet in the instrument as shown in the figure. The current through conductor, as stated, in accordance with Ampere's Right-hand Thumb Rule

	produces magnetic field B_C in a direction adding to the B_H such that modified external magnetic field affection oscillation of the magnet is $B = B_H + B_C \ldots(3)$ Magnetic field due to current carrying conductor $B_C = \frac{\mu_{0I}}{2\pi y}\ldots(4)$. Combining (3) and (4) with the given data $B = 24\times10^{-6} - \frac{4\pi\times10^{-7}}{2\pi}\times\frac{18}{0.2} \Rightarrow B = (24+18)\times10^{-6} \Rightarrow B = 42\times10^{-6}$ T…(7)  Thus, using (2) time period, when current carrying conductor is part of the system, is $T' = 2\pi\sqrt{\frac{I}{MB}}$. Accordingly, $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{I}{MB}}}{2\pi\sqrt{\frac{I}{MB_H}}} \Rightarrow T' = T\times\sqrt{\frac{B_H}{B}}$. Using the available data $T' = 0.1\times\sqrt{\frac{24}{42}} \Rightarrow T' =$ **0.076 s is the answer.**
Q-13	A short magnet makes 40 oscillations per minute when used in an oscillation magnetometer at a place when the earth's horizontal magnetic field is 25 μT. Another short magnet of magnetic moment 1.6 Am^2 is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if the magnet has its north pole (a) towards north and (b) towards south.
A-13	(a) 18 oscillations/min (b) 54 oscillations/min
I-13	Given that a short makes 40 oscillations per minute when used in an oscillation magnetometer at a place when the earth's horizontal magnetic field is $B_H = 25\times10^{-6}$ T. Therefore, time period of oscillations is $T = \frac{60}{40} = \frac{3}{2}\ldots(1)$. We know that $T = 2\pi\sqrt{\frac{I}{MB_H}}\ldots(2)$. Here, N The system is modified by introducing an external-magnet having magnetic moment $M' = 1.6$ Am^2 east of the oscillation magnetometer at a distance $y = 0.20$m in two ways as Case 1 and Case 2, as under- **Case 1:** Magnet is placed with north pole towards north as shown in the figure. It is broadside-on position. Accordingly, magnetic field produced by external-magnet at the instrument is $B_M = \frac{\mu_0}{4\pi}\times\frac{M'}{y^3}\ldots(3)$. It is seen that B_M and B_H are in opposite directions and hence net magnetic field affecting oscillations is $B'_H = B_H - B_M\ldots(4)$. Therefore, time period of oscillation would be $T' = 2\pi\sqrt{\frac{I}{MB'_H}}\ldots(5)$.  Combining (2), (3), (4) and (5), $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{I}{MB'_H}}}{2\pi\sqrt{\frac{I}{MB_H}}} \Rightarrow T' = T\times\sqrt{\frac{B_H}{B_H - \frac{\mu_0}{4\pi}\times\frac{M'}{y^3}}}$. Using the available data $T' = \frac{3}{2}\times\sqrt{\frac{25\times10^{-6}}{25\times10^{-6} - 10^{-7}\times\frac{1.6}{(0.2)^3}}} \Rightarrow T' = \frac{3}{2}\times\sqrt{\frac{25\times10^{-6}}{25\times10^{-6} - 20\times10^{-6}}}$. It solves into $T' = \frac{3}{2}\times\sqrt{5}$. Therefore, frequency of oscillations is $f' = \frac{60}{T'} = \frac{60\times2}{3\times\sqrt{5}} = 17.9$ oscillations per minute say **18 oscillations per minute is the answer of Case 1.** **Case 2:** In this case the magnet is placed with north pole towards south, as shown in the figure. It is broadside-on position. Accordingly, magnetic field produced by external-magnet at the instrument is same as in (3), with a difference, that both the B_M and B_H are in same direction and hence net magnetic field affecting oscillations is $B''_H = B_H + B_M\ldots(6)$. Therefore, time period of oscillation would be $T'' = 2\pi\sqrt{\frac{I}{MB''_H}}\ldots(7)$. 

	Combining (2), (3), (6) and (7), $\frac{T''}{T}=\frac{2\pi\sqrt{\frac{I}{MB'_H}}}{2\pi\sqrt{\frac{I}{MB_H}}} \Rightarrow T''{}' = T\times\sqrt{\frac{B_H}{B_H+\frac{\mu_0}{4\pi}\times\frac{M'}{y^3}}}$. Using the available data $T' = \frac{3}{2}\times\sqrt{\frac{25\times10^{-6}}{25\times10^{-6}+10^{-7}\times\frac{1.6}{(0.2)^3}}} \Rightarrow T'' = \frac{3}{2}\times\sqrt{\frac{25\times10^{-6}}{25\times10^{-6}+20\times10^{-6}}}$. It solves into $T'' = \frac{3}{2}\times\sqrt{\frac{5}{9}}=\frac{\sqrt{5}}{2}$. Therefore, frequency of oscillations is $f' = \frac{60}{T''}=\frac{60\times2}{\sqrt{5}}=53.7$ oscillations per minute say 54 **oscillations per minute is the answer of Case 2.** Hence, **answer is 18 oscillations per minute and 54 oscillations per minute.**
Q-14	A long straight wire carries a current i. The magnetizing field intensity H is measured at a point P close to the wire. A long, cylindrical iron rod is brought close to the wire so that the point P is at center of the rod. The value H at P will – (a) Increase many times (b) Decrease many times (c) Remain almost constant (d) Become zero
A-14	(c)
I-14	This question involves understanding of electromagnetism viz-a-viz Biot-Savart's Law. Magnitude of magnetic field intensity H at a point at a distance $\vec{r}$ from a long current carrying conductor, as per Biot-Savart's Law, is $H=\frac{i}{4\pi r}$...(1), here i is the current through the wire and is the distance of the point P close to the wire, as shown in the figure.   When a long cylindrical iron rod is brought close to the wire such that point P is at the center of the rod, the magnetizing field intensity, as shown in the figure. Iron in a ferromagnetic material and its relative permeability $\mu_r=\frac{\mu}{\mu_0}\gg 1$. Yet, expression (1) is independent of μ_r hence magnetizing field intensity will remain the same, as provided in **option (c), is the answer.**
Q-15	When ferromagnetic material goes through a hysteresis loop, the magnetic susceptibility – (a) Has no fixed value (b) May be zero (c) May be infinity (d) May be negative
A-15	(a), (b), (c), (d)
I-15	Magnetic susceptibility $\chi=\frac{I}{H}$...(1), here I is induced magnetization and H is magnetizing force. A typical hysteresis loop is shown in the figure.  • Accordingly, as per (1) susceptibility is changing at each point of the magnetization curve thus option (a) is correct. • At point D, since $H=0$ hence susceptibility is infinity making option (c) correct. • At point E, since induced magnetization is zero despite $H\neq 0$ hence option (b) is correct. • At any point of the magnetization curve in III[rd] quadrant III magnetizing force H is negative but induced magnetization I is positive hence option (d) is correct. Thus, **answer is option (a), (b), (c) and (d).**
Q-16	Assume that each iron atom has a permanent magnetic moment equal to 2 Bohr magnetization (1 Bohr magnetization equals 9.27×10^{-24} Am^2). The density of atoms in iron is 8.52×10^{28} atoms.m^{-3}. (a) Find the maximum magnetization I in a long cylinder of iron. (b) Find the maximum magnetic field B on the axis inside the cylinder
A-16	(a) 1.58×10^6 A/m (b) 2.0 T

I-16	Given that magnetic moment of each atom of iron is $M' = 2$ Bohr $= 2 \times (9.27 \times 10^{-24})$ Am^2. Density of atoms in iron $n = 8.52 \times 10^{28}$ m^{-3}. Therefore, intensity of magnetization in long cylinder of iron $I = \frac{M}{V} = \frac{(nM') \times V}{V} \Rightarrow I = nM' = (8.52 \times 10^{28}) \times \left(2 \times (9.27 \times 10^{-24})\right) \Rightarrow I = \mathbf{1.58 \times 10^{6}}$ **A/m is the answer of part (a).** Maximum magnetic field that can be achieved when all the atoms are magnetized is $B = \mu_0 H = \mu_0 I$. Since, $\frac{\mu_0}{4\pi} = 10^{-7} \Rightarrow \mu_0 = 4\pi \times 10^{-7}$, using the available data $B = (4\pi \times 10^{-7}) \times (1.58 \times 10^{6}) = \mathbf{2.0}$ **T is the answer of part (a).**

Important Note: You may encounter need of clarification on contents and analysis or an inadvertent typographical error. We would gratefully welcome your prompt feedback on mail ID: subhashjoshi2107@gmail.com. If not inconvenient, please identify yourself to help us reciprocate you suitably.

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