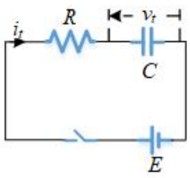
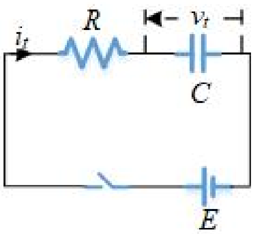
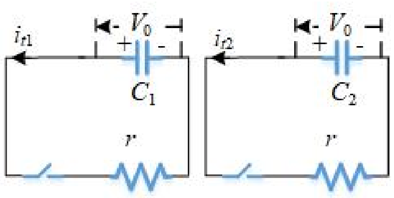
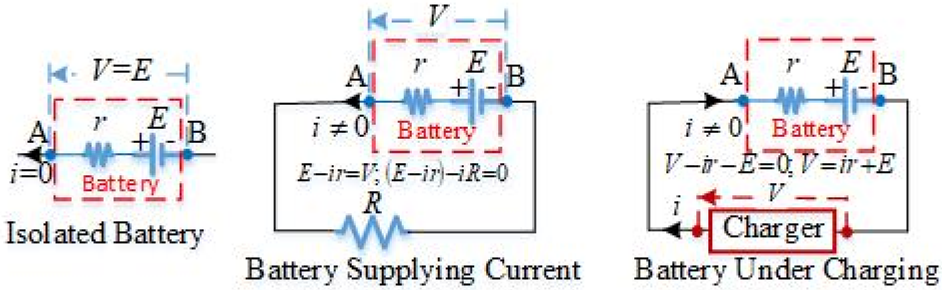
Code: Phy/ EM-2/CE/001

Electromagnetism: Current Electricity – Typical Questions (Set 1)

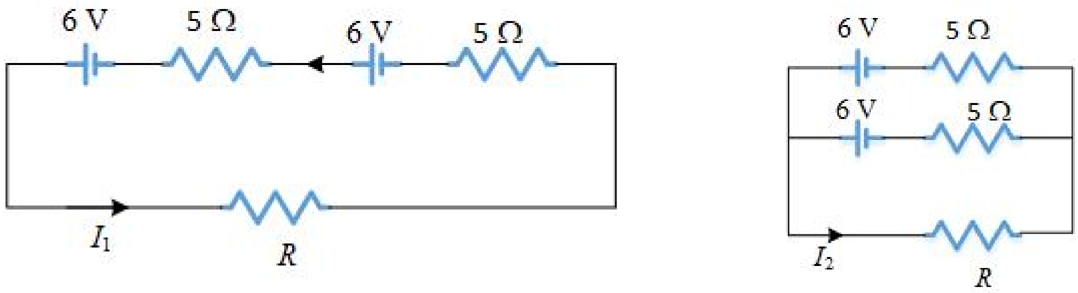
(Representative Questions Only)

Q-1	In an electrolyte, the positive ions move from left to right and negative ions from right to left. Is there a net current? If yes in which direction?
A-1	Left to right
I-1	Electric current is defined as $\vec{I} = \frac{dQ}{dt}\hat{r} \Rightarrow \vec{I} = \frac{(dN)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(dv)e_+}{dt}\hat{r} \Rightarrow \vec{I} = \frac{n(Adl)e_+}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)\frac{dl}{dt}\hat{r} \Rightarrow \vec{I} = (nAe_+)v\hat{r}$. In an electrolyte positive ions carrying charge q_+ this current has a direction left to right i.e. $\hat{r}$ and while magnitude of current due to positive ions $\vec{I}_+$ is rate of flow of charge $\frac{dQ}{dt}\hat{\imath}$. Let number of protons dN drift in time dt and hence charge drifted is $dQ_+ = (dN)q_+$. Let, n is density of positive ions a unit volume, A is cross section of beam, and dl is the distance of drift of the positive ions in time dt.Hence, $dN = n(Adl)$. Accordingly, magnitude of drift velocity of positive ions is $v = \frac{dl}{dt}$., while direction of current remains $\hat{r}$. Thus in final form of expression $(nAq_+)v$ is magnitude of current and $\hat{r}$ is the direction of current, same as the direction of positive ions from left . Accordingly, $\vec{I}_+ = (nAq_+)v\hat{r}$. Further it is given that, which is obvious, that negative ions carrying charge $q_- = (-q_+)$ this current has a direction right to left i.e. $(-\hat{r})$ and while magnitude of current due to negative ions $\vec{I}_-$ is $\frac{dQ_-}{dt} = \big(nA(-q_+)\big)v(-\hat{r}) \Rightarrow \vec{I}_- = (nAq_+)v\hat{r}$. Thus net current is $\vec{I} = \vec{I}_+ + \vec{I}_- \Rightarrow (nAq_+)v\hat{r} + (nAq_+)v\hat{r} \Rightarrow \vec{I} = 2(nAvq_+)\hat{r}$ from **left to right, is the answer.** **N.B.:** Charge on positive and negative ions in an electrolyte are equal and opposite, while average density of positive and negative ions in an electrolyte are equal.
Q-2	A fan with copper winding in its motor consumes less power as compared to an otherwise similar fan having aluminum winding. Explain
A-2	$p_{Cu} < p_{Al}$ is the reason.
I-2	Electrical power supplied to a fan is $P = VI$, whereas power loss (Power consumed in winding) in fan is $p = I^2R$. Thus power used in blowing air is $P' = P - p \Rightarrow P' = P - I^2R$. Since supply voltage V is fixed and hence for a given power of fan current is $I = \frac{P}{V}$. Accordingly, current supplied for fan having copper and aluminum winding would be same, considering all other dimensions same. Further, in a fan magnetic field $\emptyset$ is produced by current through windings such that $\emptyset \propto NI \Rightarrow \emptyset = KNI$, here K is proportionality constant and N is number of turn in the winding. Assume that length l and area of cross-section A of each turn in fan having coller and aluminum winding is same. Now resistance of a wire is $R = \frac{\rho L}{A}$. Therefore, resistance of copper winding would be $R_{Cu} = \frac{\rho_{Cu}(Nl)}{A}$ and for aluminum winding $R_{Al} = \frac{\rho_{Al}(Nl)}{A}$. Therefore, power consumed in fan having copper winding is $p_{Cu} = \left(\frac{P}{V}\right)^2 \frac{\rho_{Cu}(Nl)}{A} \Rightarrow p_{Cu} = \left(\frac{P^2Nl}{P^2A}\right)\rho_{Cu} \Rightarrow p_{Cu} \propto \rho_{Cu}$. Likewise, power consumed in fan having aluminum winding is $p_{Al} \propto \rho_{Al}$.

	It leads to $\frac{p_{Cu}}{p_{Al}} = \frac{\rho_{Cu}}{\rho_{Al}}$. We know that copper is better conductor than aluminum and hence $\rho_{Cu} < \rho_{Al}$ and hence $\boldsymbol{p_{Cu} < p_{Al}}$ **is the reason.** **N.B.:** This question involves integration of electromagnetism applied to motor action and pre knowledge of comparison on resistivity of copper and aluminum.
Q-3	A capacitor of capacitance 500 µF is connected to a battery through a 10 Ω resistor. The charge stored on the capacitor in the first 5 s is larger than the charge stored in the next (a) 5 s (b) 50 s (c) 500 s (d) 5000 s
A-3	All
I-3	 A capacitor of capacitance $C = 500$ µF is connected to a battery through a $R = 10\ \Omega$ resistor as shown in the figure. Charge stored in capacitor in time $t_1 = 5$ s, from a generic formula, is $Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow Q_5 = EC\left(1 - e^{-\left(\frac{5}{CR}\right)}\right)$…(1) Using the available data $Q_5 = EC\left(1 - e^{-\left(\frac{5}{(500\times10^{-6})10}\right)}\right) \Rightarrow Q_5 \to EC(1-0) \Rightarrow Q_5 = EC$…(2)i.e. the capacitor in the circuit gets fully charged. Therefore, for any time $t > 5$s additional charge $\Delta Q = Q_t - Q_5 = 0$ …(3)stored in the capacitor is zero. Thus, using (2) and (3) for all options are correct, is the answer. **N.B.:** This is a good problem on application of mathematical formulation of charging of capacitor.
Q-4	A capacitor C$_1$ of capacitance 1 µF and another capacitor C$_2$ of capacitance 2 µF are separately charged by a common battery for a long time. Two capacitors are then separately discharged through equal resistors. Both the discharge circuit are connected at $t = 0$. (a) The current in each of the two discharging circuit is zero at $t = 0$. (b) The current in the two discharging circuits at $t = 0$ are equal but not zero. (c) The current in the two discharging circuits at $t = 0$ are unequal. (d) C$_1$ loses 50% of its initial charge sooner than C$_2$ loses 50% of initial charge.
A-4	(b), (d)
I-4	 It is a problem on discharge of capacitor. Given that two capacitors of capacitance $C_1 = 1\times10^{-6}$ F and $C_2 = 2\times10^{-6}$ F are separately connected for a long time $t \gg$ to a common battery of emf say E. Therefore, charge on the capacitors capacitance 1 µF and another capacitor C$_2$ of capacitance 2 µF are separately charged by a common battery for a long time as shown in the figure. Thus, charge on the capacitors would be $Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow Q_t = EC$ …(1). Accordingly, $Q_1 = EC_1 \Rightarrow Q_1 = (1\times10^{-6})E$ and $Q_2 = (2\times10^{-6})E$.  These two charged capacitors are then separately discharged through equal resistors of resistance say r by connecting them $t = 0$, as shown in the figure. At this instance voltage across each is $V_0 = E = \frac{Q_1}{C_1}$. Applying Kirchhoff's Loop Law in the circuit we have $E + i_t r = 0$. It leads to $i_t r = -E \Rightarrow \frac{d}{dt}Q_t = -\frac{Q_t}{rC} \Rightarrow \frac{1}{Q_t}dQ_t = -\frac{dt}{rC}$. Integrating both sides we get $\int \frac{1}{Q_t}dQ_t = \frac{1}{rC}\int dt \Rightarrow \ln Q_t = -\frac{t}{rC} + K \Rightarrow Q_t = e^{-\left(\frac{t}{rC}+K\right)} \Rightarrow Q_t = K'e^{-\frac{t}{rC}}$…(2). Here, both K and K' are integrating constants whose value depends upon initial condition $t = 0$, such that $K' = Q_0 = EC$ ...(3). Accordingly, combining (1), (2) and (3), $Q_{1t} = EC_1 e^{-\frac{t}{rC_1}}$ and $Q_{2t} = EC_2 e^{-\frac{t}{rC_2}}$…(4). Therefore, discharge currents of two capacitors are $i_{1t} = \frac{d}{dt}Q_{1t} \Rightarrow i_{1t} = \frac{E}{r}e^{-\frac{t}{rC_1}}$.; likewise, $i_{2t} = \frac{E}{r}e^{-\frac{t}{rC_2}}$…(5).

	In this context, each of the option is being analyzed separately – **Option (a):** The current in each of the two discharging circuit at $t = 0$ are $i_{1\_0} = \frac{E}{r} e^{-\frac{0}{rC_1}} = \frac{E}{r}$ and likewise $i_{2\_0} = \frac{E}{r} \Rightarrow i_{1\_0} = i_{2\_0} = \frac{E}{r}$. Thus, this option suggesting $i_{1\_0} = i_{2\_0} = 0$ **is wrong.** **Option (b):** Taking forward analysis in option (a), $i_{1\_0} = i_{2\_0} = \frac{E}{r} \neq 0$. Thus, **this option is correct.** **Option (c):** Unequal currents in the two discharging circuits at $t = 0$ in light conclusion at (b), **this option is wrong.** **Option (d):** Let t_1 is the time taken by C_1 to lose loses 50% is $Q_{1t_1} = 0.5 \times Q_{1_0}$is, as per (4), $0.5 = e^{-\frac{t_1}{rC_1}}$. …(6). Likewise, for capacitor C_2 in time t_1, $0.5 = e^{-\frac{t_2}{rC_2}}$…(7). Combining (6) and (7), $e^{-\frac{t_1}{rC_1}} = e^{-\frac{t_2}{rC_2}}$. Applying, theory of indices $\frac{t_1}{C_1} = \frac{t_2}{C_2} \Rightarrow t \propto C$. Accordingly, with the given $C_1 < C_2$ we have $t_1 < t_2$. Thus, **this option is correct.** **Thus, answer is options (b) and (d).** **N.B.:** This problem involves solution of linear differential equation of first order formulated for charge and discharge-current on a discharging capacitor, and is slightly different than charging of capacitor.
Q-5	The potential difference between the terminals of a 6.0 V battery is 7.2 V when it is being charged by a current of 2.0 A. What is the internal resistance of the battery?
A-5	0.6 Ω
I-5	When a battery of emf E is charged an external source of potential difference $V > E$ is connected with reverse polarity across the battery to pump current in the battery. Accordingly, the circuit equation is $V - Ir - E = 0 \Rightarrow r = \frac{V-E}{I}$. Using the available data $r = \frac{7.2-6.0}{2.0} \Rightarrow r = \mathbf{0.6\ \Omega}$ **is the answer.** **N.B.:** Circuit equation for a battery under charging needs to be noted carefully since in this case current is pumped into the battery causing a potential difference, higher than its emf, across it. Whereas when battery is supplying current potential difference across the battery is lower than its emf. The concept is shown in the figure. 

Q-6 Find the value of $\frac{i_1}{i_2}$ in figure if (a) $R = 0.1\ \Omega$, (b) $R = 1\ \Omega$, (c) $R = 10\ \Omega$. Note from your answer that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if external resistance is large as compared to the internal resistances.



A-6 (a) 0.51 (b) 1 (c) 1.25

I-6 A close examination of circuits reveal that in first circuit the two batteries each of emf $E = 6$ Vare connected in series with net emf in the circuit $E_1 = E + E \Rightarrow E_1 = 2E$, and hence their internal resistances $r = 5\ \Omega$ will also be in series $r_1 = r + r = 2r$. In turn supplying current to external resistance R. Thus, $i_1 = \frac{E_1}{R+r_1} \Rightarrow i_1 = \frac{2E}{R+2r}$...(1).

In second circuit both the batteries are connected in parallel and hence emf in the circuit is $E_2 = E$, while potential difference across external resistance is $V_2 = i_2 R$. The internal resistances of the two batteries shall also be in parallel such that $r_2 = \frac{r\times r}{r+r} \Rightarrow r_2 = \frac{r}{2}$. Hence, current in the circuit $i_2 = \frac{E}{R+\frac{r}{2}} \Rightarrow i_2 = \frac{2E}{2R+r}$...(2). *Expression in this case can be verified by applying Kirchhoff's Loop Law also.*

Accordingly, the desired ratio using (1) and (2) is $\frac{i_1}{i_2} = \frac{\frac{2E}{R+2r}}{\frac{2E}{2R+r}} \Rightarrow \frac{i_1}{i_2} = \frac{2R+r}{R+2r}$...(3). Using this final form with the available data for each of the value of R –

Case (a): $R = 0.1\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 0.1+5}{0.1+2\times 5} = 0.51$

Case (b): $R = 1\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 1+5}{1+2\times 5} = 1$

Case (c): $R = 10\ \Omega$, using the available data the ratio $\frac{i_1}{i_2} = \frac{2\times R+r}{R+2\times r} \Rightarrow \frac{i_1}{i_2} = \frac{2\times 10+5}{10+2\times 5} = 1.25$.

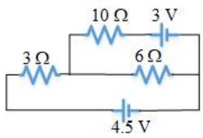
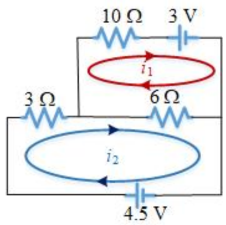
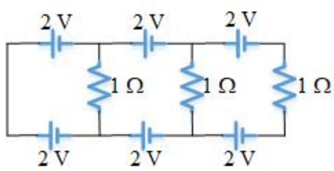
N.B.: Solving problem algebraically and substituting numerical values at last stage greatly simplifies calculation. All that it needs is proficiency and patience to handle algebraic expressions.

Q-7 A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance R. If R is not greater than 100 Ω, the current through it is constant up to two significant digits. Find its value. This is the basic principle of a *constant-current source.*

A-7 10 mA

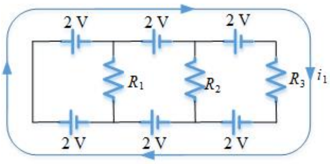
I-7 The system can be considered as a battery of emf $E = 100$ V and internal resistance $r = 10 \times 10^3$ W supplying current to an external resistance $R \leq 100\ \Omega$.. Thus current supplied by the battery is $I = \frac{E}{R+r}$. In this since is in the denominator and applying the specified constraint $I = \frac{100}{100+10\times 10^3} \Rightarrow I = \frac{1}{101} \approx 10$ mA and if $R = 0$ the current is $I = \frac{100}{0+10\times 10^3} \Rightarrow I = \frac{1}{100} \approx 10$ mA. Thus constant current upto two SDs is **10 mA is the answer.**

N.B.: (a) This is a good example to numerically understand how constant current source of small current can be achieved.

	(b) In this a large r in series with battery is considered as an internal resistance of battery is only a circuit analogy for analysis. In fact internal resistance is of small value.
Q-8	Three bulbs, each having a resistance of 180 Ω are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when – (a) All the bulbs a switched on, (b) Two of the bulbs are switched on, (c) Only one bulb is switched on.
A-8	(a) 1.0 A, (b) 0.67 A, (c) 0.33 A
I-8	Three bulbs each of resistance $R = 180$ W are connected in parallel across a battery of $V = 60$ V and can be switched on selectively, in three ways are as under – **Way (a):** All bulbs are switched on hence equivalent resistance would be $R_a = \frac{R}{3}$, therefore current $I_a = \frac{V}{R_a} \Rightarrow$ $I_a = \frac{60}{\frac{180}{3}} = \mathbf{1.0}$ **A, is the answer**. **Way (b):** Two bulbs are switched on hence equivalent resistance would be $R_b = \frac{R}{2}$, therefore current $I_b = \frac{V}{R_b} \Rightarrow I_b = \frac{60}{\frac{180}{2}} = \mathbf{0.67}$ **A, is the answer**. **Way (c):** Only one bulbs is switched on hence equivalent resistance would be $R_c = R$, therefore current $I_c = \frac{V}{R_c} \Rightarrow I_b = \frac{60}{180} = \mathbf{0.33}$ **A, is the answer**. Thus, **answers are (a) 1.0 A, (b) 0.67 A, (c) 0.33 A** **N.B.:** Principle of SDs is used while reporting the answer.
Q-9	Find the current through the 10 Ω resistor shown in figure. 
A-9	Zero
I-9	The given circuit is redrawn by identifying two loops with respective currents i_1 and i_2 as shown in the figure. Writing loop equations as per Kirchhoff's Circuit Law as under – **Loop 1:** Loop current is i_1; $-3-(i_1-i_2)6-i_1\times 10=0 \Rightarrow -16i_1+6i_2=3\ldots(1)$. **Loop 2:** Loop current is i_2; $4.5-i_2\times 3-(i_2-i_1)6=0 \Rightarrow 6i_1-9i_2=-4.5\ldots(2)$.  To solve these equations $(1)\times 3+(2)\times 2$ leads to $(-48+12)i_1=0$. This $i_1=0$ A is the answer. **N.B.:** Simple application of Kirchhoff's Law which require cares in using sign convention as under- **2)** Generally each loop is identified with a loop current. Emf of all batteries (or voltage sources) with polarities aligned in the direction of the loop current is taken to be (+)ve. Thus, emf of batteries with polarities against direction of loop current is assigned (-)ve. **3)** In a resistor, loop current of the loop for which KLL equation being written is taken (+)ve and any other current in the resistor is assigned sign relative to the loop current. **4)** Potential difference across a resistor, with the consideration of current through it as per (2) above is assigned (-)ve.
Q-10	Find the current in the three resistors shown in figure. 

A-10 Zero

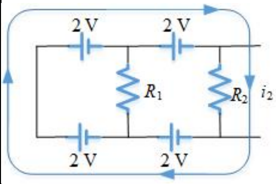
I-10 The given circuit, with each resistance $R_1 = R_2 = R_3 = 1\ \Omega$, as shown in the figure, is analyzed in three stages using Kirchhoff's Loop Law as under –



Loop current i_1: It has only one resistance with six batteries each of 2 V, three in the upper line are in direction opposite to the loop current, while the three in bottom in direction of the loop current. Accordingly, the loop equation is:

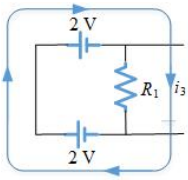
$$(-2) + (-2) + (-2) - i_1R_3 + 2 + 2 + 2 = 0 \Rightarrow$$

$\boldsymbol{i_1 = 0}$.



Loop Current i_2: It has Four batteries, two in the upper line are in direction opposite to the loop current, while the two in bottom in direction of the loop current. Accordingly, the loop equation is:

$(-2) + (-2) - i_2R_2 + 2 + 2 = 0 \Rightarrow \boldsymbol{i_2 = 0}$.



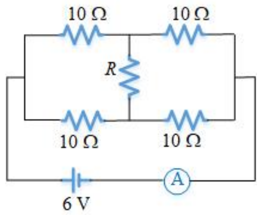
Loop Current i_3: It has tour batteries, one in the upper line are in direction opposite to the loop current, while the other in bottom in direction of the loop current. Accordingly, the loop equation is:

$(-2) - i_3R_1 + 2 = 0 \Rightarrow \boldsymbol{i_3 = 0}$.

Thus, answer is Zero.

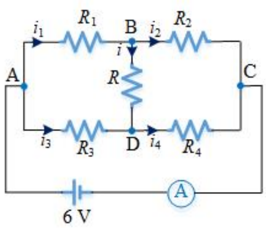
N.B.: A close observation of the given circuit has been simplified with simple equation of external loops, instead of considering internal loops. It is specific to the problem and cannot be taken as a general rule.

Q-11 What should be the value of R in figure for which the current in it is zero?



A-11 Any value of R will do

I-11 The given circuit is redrawn, in a generic sense, with resistances of $10\ \Omega$ shown algebraically such that $R_1 = R_2 = R_3 = R_4 = 10\ \Omega$, with branch currents i_1, i_2, i_3, i_4 and i, and nodes A,B,C and D identified, as shown in the figure. It is also given that current $i = 0 \ldots(1)$, through resistance R. With this this value of R is to be determined.



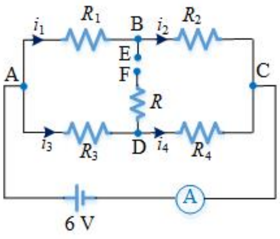
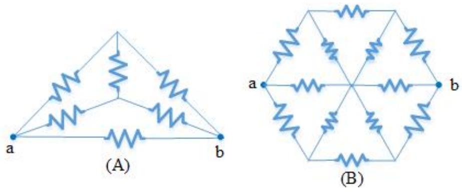
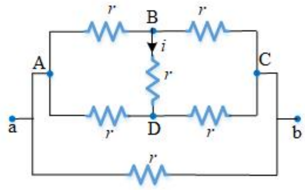
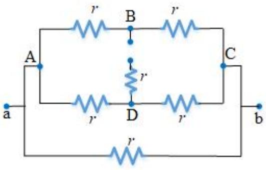
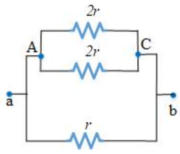
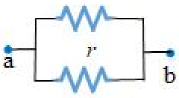
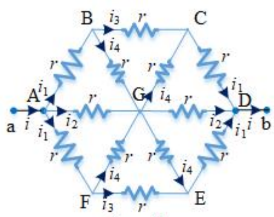
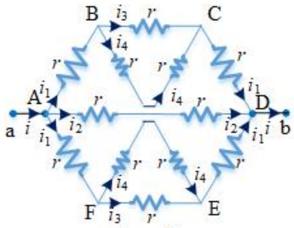
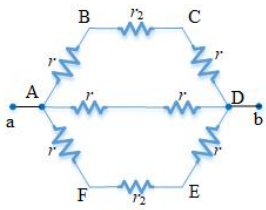
The current i through R, as per Ohm's Law, is $i = \frac{V_B - V_D}{R} \ldots(2)$, considering assigned direction to the current from node B to D. Combining (1) and (2), $\frac{V_B - V_D}{R} = 0$ is possible when - **(a)** Either $V_B - V_D = 0 \ldots(3)$, or **(b)** $R \to \infty$.

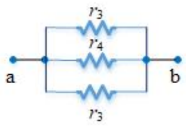
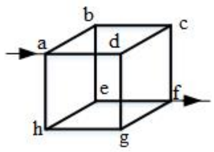
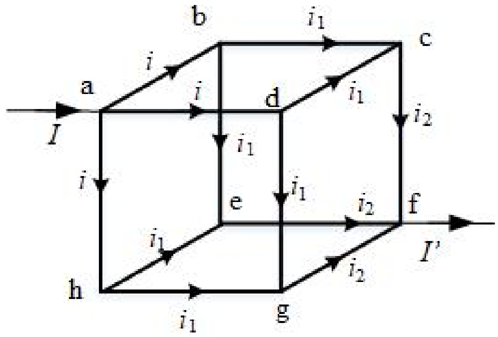
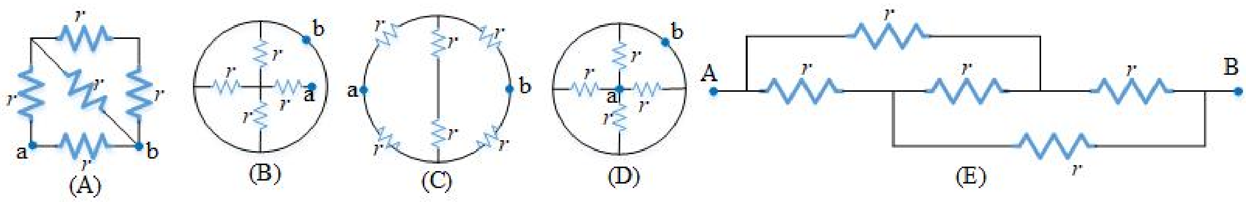
Taking possibility (a) and applying voltage equation to branches AB, BC, AD and DC we have –

$V_A - V_B = i_1R_1 \ldots(4)$, $V_A - V_D = i_3R_3 \ldots(5)$, $V_B - V_C = i_2R_2 \ldots(6)$, and $V_D - V_C = i_4R_4 \ldots(7)$.

And as per Kirchhoff's current law at node B $i_1 = i + i_2$; using (1) $i_1 = i_2 \ldots(8)$., and at node D $i_3 + i = i_4$, again using (1) $i_3 = i_4 \ldots(9)$.

Using (3), (4) and (5), $V_B - V_D = i_1R_1 - i_3R_3 = 0 \Rightarrow \frac{i_1}{i_3} = \frac{R_3}{R_1} \ldots(10)$. Likewise, using (3), (6) and (7), we have $V_B - V_D = i_2R_2 - i_4R_4 = 0 \Rightarrow \frac{i_2}{i_4} = \frac{R_4}{R_2} \ldots(11)$.

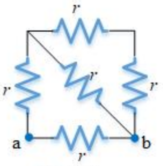
	 Now combining (11) with (8) and (9), $\frac{i_1}{i_3}=\frac{R_4}{R_2}$…(12). Comparing (10) and (12) $\frac{R_3}{R_1}=\frac{R_4}{R_2}\Rightarrow\frac{R_1}{R_2}=\frac{R_3}{R_4}$. …(13). Using the given values of resistances $\frac{R_1}{R_2}=\frac{R_3}{R_4}=\frac{10}{10}=1$ for $i=0$ as given at (1). This is similar to absence of connection between B and D, i.e. open circuit with nodes E and F, as shown in the figure. Therefore, **any value of R will do** is the answer. **N.B.:** The given network is different graphical representation of Wheatstone Bridge, Appendix-II. It is a ***very important circuit*** extensively used for different application in different names. It was developed by Samuel Hunter Christie (year 1833) and modified by Sir Charles Wheatstone year 1943) and known after his name.
Q-12	Find equivalent resistance of the circuit shown in figure between points a and b. Each resistor has a resistance r. 
A-12	(a) $\frac{r}{2}$, (b) $\frac{4r}{5}$
I-12	Each of the circuit is being analyzed separately, **Circuit A:** Electrical equivalent of the circuit is shown in the figure. It is seen that network between nodes A-B-C-D-A is a Wheatstone bride where $\frac{R_{AB}}{R_{AD}}=\frac{R_{BC}}{R_{DC}}=1$, elaborated in Appendix-I. Thus it satisfies condition current though branch B-D is $i=0$. it is analogues to branch an open circuit as shown in the figure. Thus it simplifies into a circuit as shown in the figure taking series resistances of branches A-B-C and A-D-C. Next step is equivalent resistance of two parallel resistors of resistances $2r$. It further simplifies into $\frac{2r\times2r}{2r+2r}=r$. In its last form it is parallel combination of two resistors of resistance r as shown in the circuit. Thus, the network reduces to an equivalent resistance $R=\frac{r\times r}{r+r}=\frac{r}{2}$ **is the answer.**     **Circuit B:** The network of resistances at a first glance is combination of delta-connection. But, all resistance being equal to r adds symmetry to the network both about a-b and central line perpendicular to a-b and passing through G. This symmetry simplifies problem instead of using Δ-Y transformations as per Appendix-III. Accordingly, currents through different branches have been indicated in figure of stage 1.  Stage 1  Stage 2 Considering mirror symmetry of the currents as shown in the figure which is – (i) current i_2 entering node G in branch AG is equal to current leaving the node in branch GD, (ii) Current i_4 entering node G through branches BG and FG would be equal to currents leaving the node through branches GC and GE. These observations have been utilized to isolate upper delta and lower delta connections having vertex at G as shown ion the figure of stage 2. Thus results into parallel connection between nodes BC and FE such that $r_2=\frac{r\times2r}{r+2r}\Rightarrow r_2=\frac{2r}{3}$ as shown in the network of stage 3.  Stage 3

	Thus network reduces to parallel combination of three branches ABCD and AFED, each having equivalent resistances $r_3 = r + \frac{2r}{3} + r \Rightarrow r_3 = \frac{8r}{3}$. And AD having equivalent resistance $r_4 = r + r \Rightarrow r_4 = 2r$. Thus, further reduced network is shown in figure of stage 4 having three resistances r_3, r_4 and r_3 in parallel. Thus equivalent resistance R of the network between nodes a and b is $\frac{1}{R} = \frac{1}{r_3} + \frac{1}{r_4} + \frac{1}{r_3} = \frac{1}{\frac{8r}{3}} + \frac{1}{2r} + \frac{1}{\frac{8r}{3}} \Rightarrow \frac{1}{R} = \frac{3}{8r} + \frac{1}{2r} + \frac{3}{8r} = \frac{10}{8r} \Rightarrow R = \frac{4r}{5}$  Stage 4 **N.B.:** Such, network problems require systematic use of series-parallel equivalent resistances, Wheatstone Bridge wherever applicable and Star-Delta equivalent networks, discussed in Appendix II and III. This approach is useful to solve any complex network without forming long node and loop equation as per Kirchhoff's Law and Ohm's Law. These two basic laws are the foundation of network analysis, all that it needs is a patience to analyze the network.
Q-13	Twelve wires, each having equal resistance r are joined to form a cube as shown in the figure. Find the equivalent resistance between the diagonally opposite points a and f. 
A-13	$\frac{5}{6}r$
I-13	Equivalent resistance of the given network is being analyzed considering its electrical symmetry and accordingly distribution of current through various branches of the network as shown in the figure. Thus, writing Kirchhoff's Node Equation at different nodes –  **Node a:** $I - i - i - i = 0 \Rightarrow 3i = I \Rightarrow i = \frac{I}{3}$ …(1). **Nodes b, d and h:** $i - i_1 - i_1 = 0 \Rightarrow 2i_1 = i \Rightarrow i_1 = \frac{i}{2} \Rightarrow i_1 = \frac{\frac{I}{3}}{2} \Rightarrow i_1 = \frac{I}{6}$…(2). **Nodes c, g and e:** $i_1 + i_1 - i_2 = 0 \Rightarrow i_2 = 2i_1 \Rightarrow i_2 = 2 \times \frac{I}{6} \Rightarrow i_2 = \frac{I}{3}$…(3) **Node f:** $i_2 + i_2 + i_2 - I' = 0 \Rightarrow I' = 3 \times \frac{I}{3} = I' \Rightarrow I$…(4) The shortest path from node a to f having unidirectional currents are (i) a-d-g-f, (ii) a-d-c-f, (iii) a-b-c-f, (iv) a-b-e-f, (v) a-h-g-f and (vi) a-h-e-f. Given Each branch of the network is of a wire of has resistance r. Utilizing the network symmetry as observed voltage equation is $V_a - ir - i_1 r - i_2 r - V_f = 0$. Using equations (1), (2) and (3), $\Delta V = V_a - V_f = (i + i_1 + i_2)r \Rightarrow \Delta V = \left(\frac{I}{3} + \frac{I}{6} + \frac{I}{3}\right)r \Rightarrow \Delta V = \frac{5}{6}Ir$…(5) Using (4), considering equivalent resistance between points a and f to be R we have $\Delta V = IR$…(6). Combining (5) and (6), $IR = \frac{5}{6}Ir \Rightarrow R = \boldsymbol{\frac{5}{6}r}$ **is the answer**. **N.B.:** Combing geometrical symmetry and electrical symmetry, with careful observation of the given network greatly simplifies the solution.
Q-14	Find the equivalent resistances of the networks shown in figure between points a and b. 
A-14	(a) $\frac{5}{8}r$, (b) $\frac{4}{3}r$, (c) r, (d) $\frac{r}{4}$, (e) r

I-14

It is required to equivalent resistance R between nodes a and b, in the given five different combinations of resistors of resistances r shown in figure A, B, C, D and E. Each of the combination is being solve, in a stage-wise manner, independently using series, parallel and star-delta equivalent as per need. Simplification of network in each stage is shown in respective figure –

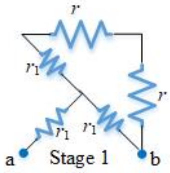
Figure A:



Stage 0

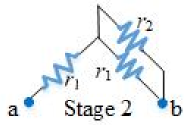
Stage 0: It is the given network in which lower lower electrically similar Δ is selected for its Y equivalent. This choice is based on consideration that each of the equivalent resistance remains part of the network. Whereas upper delta will leave one of the star limb open in the network. Accordingly, symmetrical star equivalent resistance, as per Appendix-III is $r_1 = \frac{r}{3}$. It is taken forward in stage 1 resolution of network, as under.

Stage 1: In this upper triangular connection is not delta rather it is a parallel combination of resistance $r_1$with a series combination of resistances $r_1 + r + r = \frac{r}{3} + 2r \Rightarrow r_2 = \frac{7r}{3}$. This equivalent is used in stage 2 resolution of network, as under.



Stage 1

Stage 2: Parallel combination of resistances r_1 and r_2 in this stage is $r_3 = \frac{r_1 \times r_2}{r_1 + r_2} = \frac{\left(\frac{r}{3}\right)\times\left(\frac{7r}{3}\right)}{\frac{r}{3}+\frac{7r}{3}} \Rightarrow r_3 = \frac{7r}{24}$. This equivalent is used in stage 2 resolution of network, as under.



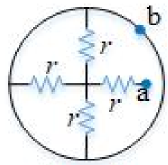
Stage 2

Stage3: In final stage it is a series combination of resistances r_1 and r_2. Thus equivalent resistance of the given combination is is $R = r_1 + r_3 \Rightarrow R = \frac{r}{3} + \frac{7r}{24} = \frac{15r}{24} \Rightarrow R = \frac{5r}{8}$ **is the answer.**

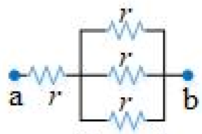
Stage 3

Figure B:

Stage 0: A close observation of given connection of resistors reveal that **e**lectrically it contains a parallel combination of three resistors of resistance r as shown in stage 1.



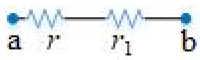
Stage 0



Stage 1

Stage 1: Equivalent resistance of the parallel combination is $r_1 = \frac{r}{3}$. It is taken forward in stage 2 resolution of network, as under.

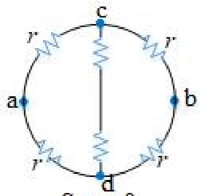
Stage 2: As shown in the figure it is a series combination of two resistances r and r_1. Thus equivalent resistance of the given combination is of the parallel combination is $R = r + r_1 \Rightarrow R = r + \frac{r}{3} \Rightarrow R = \frac{4r}{3}$ **is the answer.**



Stage 2

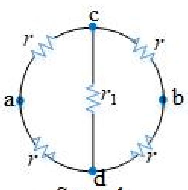
Figure C:

Stage 0: Given network has a series combination of two resistors of resistance r connected along diameter between nodes marked c and d. Thus, $r_1 = r + r = 2r$, as shown in the figure in stage 1.



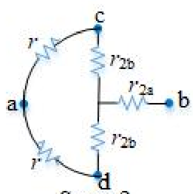
Stage 0

Stage 1: In the reduced network of resistances any of the two identical geometrical half can be taken as Δ connection of resistances. Accordingly, choice is made of right-half for leading to equivalent resistance. It is to be seen that it is not an equilateral delta as resistances between nodes b-c and b-d are r while resistance between nodes c-d is r_1. Thus, equivalent resistances of star-equivalent as per Appendix-III is $r_{2a} = \frac{r \times r}{r + r + r_1} = \frac{r \times r}{r + r + 2r} \Rightarrow r_{2a} = \frac{r}{4}$, while $r_{2b} = \frac{r \times r_1}{r + r + r_1} = \frac{r \times 2r}{r + r + 2r} \Rightarrow r_{2b} = \frac{r}{2}$. It is taken forward in stage 2 for resolution of network, as under.

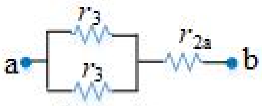


Stage 1

Stage 2: It has two identical series combination of resistances r and r_{2b} such that $r_3 = r + r_{2b} = r + \frac{r}{2} \Rightarrow r_3 = \frac{3r}{2}$. It is used in stage 2 for solving network, as under.



Stage 2



Stage 3

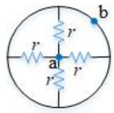
Stage 3: It has a parallel combination of two identical resistors r_3 such that $r_4 = \frac{r_3}{2} = \frac{\frac{3r}{2}}{2} \Rightarrow r_4 = \frac{3r}{4}$. It is used in stage 2 for solving network, as under.

Stage 4: It is the last stage having series combination of two identical resistors r_4 and r_{2a}. Thus equivalent resistance is $R = r_4 + r_{2a} = \frac{3r}{4} + \frac{r}{4} \Rightarrow R = \boldsymbol{r}$ **is the answer**.

Stage 4

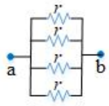
Figure D:

Stage 0: A close observation of the geometry of the network reveals that one end of all four resistors is connected to node a. Whereas, the other end of all the four resistances is electrically connected to node b. This is shown in the figure in stage 1.



Stage 0

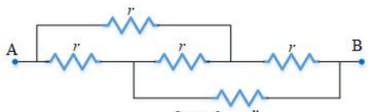
Stage 1: Thus, parallel combination of four resistors of resistance $r_1 = \frac{r}{4}$. There is no other resistance between nodes a and b. Hence equivalent resistance is $R = \boldsymbol{\frac{r}{4}}$ **is the answer.**



Stage 1

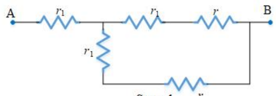
Figure E:

Stage 0: The given network has two symmetrical Δ-connection of resistors one with resistance r above the three identical resistors of resistance r directly between A and B and the other with identical resistance below the line A-B. Any of Δ-connection can be converted into Y-connection, as per Appendix-III. Choice is made of Δ-connection with the resistance r above the line AB, $r_1 = \frac{r}{3}$. This is used in stage 1.



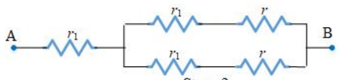
Stage 0

Stage 1: As shown in the figure which has a parallel combination of two branches of identical resistances connected in series. This connection for clarity is redrawn in stage 2.

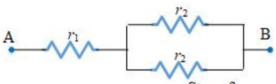


Stage 1

Stage 2: Equivalent resistance of each of the branch of parallel combination is $r_2 = r_1 + r = \frac{r}{3} + r = \frac{4r}{3}$. It is used in stage 3 for further reducing the network.



Stage 2



Stage 3

Stage 3: The parallel combination of two resistors of resistance r_2 is $r_3 = \frac{r_2}{2} \Rightarrow r_3 = \frac{\frac{4r}{3}}{2} \Rightarrow r_3 = \frac{2r}{3}$. This is taken forward in last stage of the simplification in stage 4.

Stage 4: As shown in the figure it is a series combination of resistances r_1 and r_3, and finally $R = r_1 + r_3 = \frac{r}{3} + \frac{2r}{3} \Rightarrow R = \boldsymbol{r}$ **is the answer.**

Stage 4

Thus, **answers are** (a) $\frac{5}{8}r$, (b) $\frac{4}{3}r$, (c) r, (d) $\frac{r}{4}$, (e) r

N.B.: 1. Geometrical connection of resistors is at times to test conceptual clarity in respect of electrical connection.

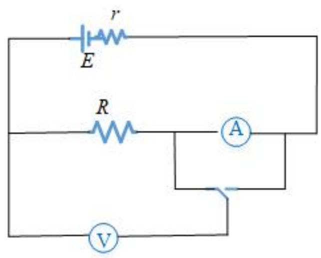
2. Given set of Five connections tries to combine different types of connections and solving each of them will help to develop confidence in determining equivalent resistance of a network of resistances.

3. ***Patience is the key*** *in successfully handling problems of electricity* during studies. It is an opportunity to develop patience required in practice in real life, specially involving electricity, which otherwise could lead to dangerous experiences.

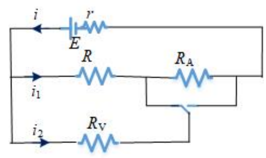
<table>
<tr><td>Q-15</td><td>
The emf E and the internal resistance r of the battery shown in the figure are 4.3 V and 1.0 Ω respectively. The external resistance R is 50 Ω. The resistances of the ammeter and voltmeter are 2.0 Ω and 200 Ω respectively.

(a) Find the reading of the two meters.

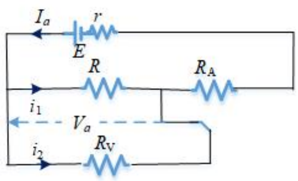
(b) The switch is thrown to the other side. What will be the reading of the two meters now ?



</td></tr>
<tr><td>A-15</td><td>(a) 0.1 A, 4.0 V, (b) 0.08 A, 4.2 V</td></tr>
<tr><td>I-15</td><td>
Given is a circuit with significant resistances of ammeter $R_A = 2.0\ \Omega$ and voltmeter $R_V = 200\ \Omega$, respectively. These resistances are inherent resistances of the two meters having a current sensing device. Therefore, reading of ammeter and voltmeter is proportional to current flowing through it and in turn it is dependent on voltage across the meter. The circuit having battery of emf $E = 4.8$ V and internal resistance $r = 1.0\ \Omega$ is connected with the two meters to measure current and voltages about an external resistance $R = 50\ \Omega$. The circuit is redrawn, as shown in the figure, with the meters replaced by respective resistances.

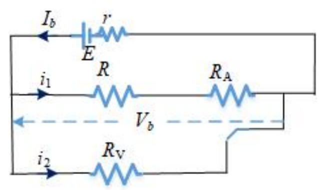


Accordingly, the given circuit to measure voltage and current about resistor R in part (a) and about resistor $R + R_A$ in part (b). These voltages and currents are calculated using circuit equations of each part separately.



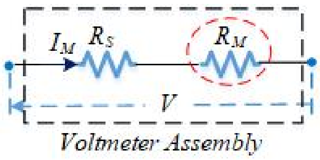
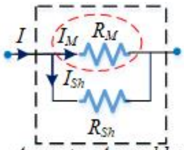
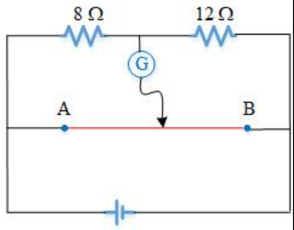
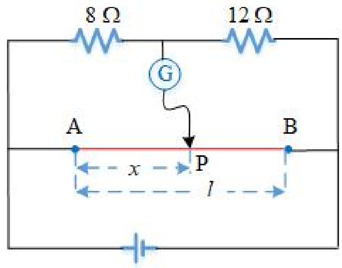
Part (a): In this R and R_V form parallel combination which is in series with r and R_A. Thus equivalent resistance of the circuit is $R_a = \frac{R \times R_V}{R + R_V} + R_A + r \Rightarrow R_a = \frac{50 \times 200}{50 + 200} + 2.0 + 1.0 \Rightarrow R_a = 43\ \Omega$. Therefore, circuit current is $I_a = \frac{E}{R_a} = \frac{4.3}{43} = \mathbf{0.10}$ A. Though the circuit current branches into i_1 and i_2 in resistors R and R_V, yet current through ammeter remains I_a and therefore the ammeter would read $I_a = 0.11$ A. As regards voltmeter, it will read the voltage across parallel combination of R and R_V equal to $V_a = E - I_a \times (r + R_A) = 4.3 - 0.1(1.0 + 2.0) \Rightarrow V_a = 4.0$ V, say 4.0 V.

Part (b):This places voltmeter across resistance R and ammeter resistance R_A. Thus, equivalent resistance of the circuit changes to $R_b = \frac{(R + R_A) \times R_V}{(R + R_A) + R_V} + r \Rightarrow R_b = \frac{(50 + 2.0) \times 200}{(50 + 2.0) + 200} + 1.0 \Rightarrow R_b = 41.3\ \Omega$. Therefore, circuit current will be also $I_b = \frac{4.3}{41.3} = 0.10$ A. Accordingly, voltage across the voltmeter would be $V_b = E - I_b r = 4.3 - 0.12 \times 1.0 \Rightarrow V_b = 4.18$ V, say 4.2 V As regards current $I_b = i_1 + i_2$, hence i_1 and i_2, in parallel branches are in inverse proportion of their resistances, $\frac{i_2}{i_1} = \frac{R + R_A}{R_V}$. It leads to $\frac{i_2 + i_1}{i_1} = \frac{R + R_A + R_V}{R_V} \Rightarrow i_1 = \frac{I_b R_V}{R + R_A + R_V} = \frac{0.10 \times 200}{50 + 2.0 + 200} \Rightarrow i_1 = 0.079$ A say 0.08 A.



Thus, answers are (a) 0.10 A, 4.0 V (b) 0.08 A, 4.2 V.

N.B.: This example nicely highlights impact of meter resistance and connected resistance on meter reading.
</td></tr>
<tr><td>Q-16</td><td>A voltmeter coil has resistance 50.0 Ω and a resistor of 1.15 kΩ is connected in series. It can read potential difference upto 12 volts. If this same coil is used to construct an ammeter which measure current upto 2.0 A, what should be the resistance of the shunt used?</td></tr>
<tr><td>A-16</td><td>0.251 Ω</td></tr>
</table>

I-16	Voltmeter is an assembly of a current sensing device having a resistance defined as that of its coil given to be $R_M = 50.0\ \Omega$ with a series resistance R_S. In the instant case given that with $R_S = 1.15 \times 10^3\Omega$ to read maximum voltage $V = 12$ V. Thus, maximum current through meter is $I_M = \frac{V}{R_M+R_S} = \frac{12}{50.0+1.15\times10^3} \Rightarrow I_M = 0.01\text{A}\ldots(1)$.  Voltmeter Assembly  Ammeter Assembly The same coil, meaning the same current sensing device, is used to measure maximum current $I = 2.0$ A. This is achieved by using a shunt of resistance R_{Sh} in parallel to the device as shown in the figure. This causes branching of current into I_M and I_{Sh} through R_M and R_{Sh} such that $I = I_M + I_{Sh}\ldots(2)$. In parallel combination $\frac{I_M}{I_{Sh}} = \frac{R_{Sh}}{R_M} \Rightarrow \frac{I_M}{I_M+I_{Sh}} = \frac{R_{Sh}}{R_{Sh}+R_M} \Rightarrow I_M(R_{Sh} + R_M) = IR_{Sh} \Rightarrow (I - I_M)R_{Sh} = I_M R_M \Rightarrow R_{Sh} = \frac{I_M R_M}{I-I_M}$. Using the available data $R_{Sh} = \frac{0.01\times50.0}{2.0-0.01} = \mathbf{0.251\Omega}$ **is the answer.**
Q-17	The potentiometer wire AB shown in figure is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection? 
A-17	16 cm from A
I-17	Let a point P is a distance x from end A of the potentiometer wire, then distance of wire between P and B is $l - x$. Potentiometer works on principle of Wheatstone bridge at its null point of zero-deflection.  Let resistance per unit length of the potentiometer wire is ρ Ω/cm. Therefore, resistance between A-P is $R_1 = \rho x$ and resistance between P-B is $R_2 = \rho(l - x)$. Likewise, $R_3 = 8\ \Omega$ and $R_4 = 12\ \Omega$. Thus at null point $\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \frac{\rho x}{\rho(l-x)} = \frac{8}{12} \Rightarrow \frac{x}{x+(40-x)} = \frac{8}{8+12} \Rightarrow \frac{x}{40} = \frac{8}{20}$. It solves into $x = 40 \times \frac{8}{20} \Rightarrow x =$ **16 cm is the answer.**

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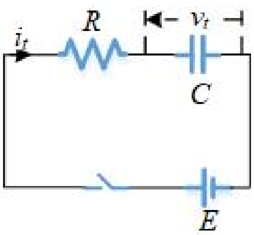
Appendix-I

Charging and Discharging of a Capacitor Through Resistance

Charging and discharging of a capacitor-charging in current electricity is dynamic application of electrostatics. While basics concept remains same initial condition of capacitor gives different results for both the processes. Accordingly, they are being discussed separately –

Charging of a Capacitor: A capacitor charging circuit is shown in the figure. The capacitor is initially discharged i.e. at $t = 0$ charge on capacitor is $Q_t = 0$, accordingly voltage across capacitor at the instant is $Q_t = Cv_t \Rightarrow v_0 = \frac{Q_o}{C} = 0$.



After closing the switch at any instant t, as per Kirchhoff's Loop Law in the circuit we have $E - i_t R - v_t = 0 \Rightarrow E - v_t = i_t R \Rightarrow E - \frac{Q_t}{C} = \frac{dQ_t}{dt}R \Rightarrow dQ_t = \left(\frac{EC-Q_t}{CR}\right)dt\ \ldots(1)$. Here, $i_t$is capacitor charging current at any instant t.

Equation (1) is a linear differential equation and its solution is $\int \frac{dQ_t}{EC-Q_t} = \frac{1}{CR}\int dt + K\ldots(2)$ here K is an integration constant whose value can be determined based on initial condition on solution of

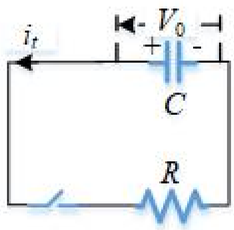
(2). Substituting $u = EC - Q_t \Rightarrow du = -dQ_t$ in (2) we have $-\int \frac{du}{u} = \frac{t}{CR} + K \Rightarrow -\ln u = \frac{t}{CR} + K$…(3). Reversing the substitution in (3) $\ln(EC - Q_t) = -\frac{t}{CR} - K \Rightarrow EC - Q_t = K'e^{-\left(\frac{t}{CR}\right)}$…(4) here $K' = e^{-K}$ is another form of the constant K.

Using the initial condition in (4), $EC - 0 = K'e^{-\left(\frac{0}{CR}\right)} \Rightarrow K' = EC$…(5). Combining (4) and (5), $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$…(6). Accordingly, charging current is $i_t = \frac{d}{dt}Q_t = \frac{d}{dt}EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right) \Rightarrow \boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$ …(7).

Finally, during charging of a capacitor, at any instant t , charge $\boldsymbol{Q_t}$ on it and current through the resistor $\boldsymbol{i_t}$ is-

(A) $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$ (B) $\boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$

Discharging of a Capacitor: It is with an initial state of capacitor when it carries some charge. But, in this simplified case no voltage source is considered in the circuit, as shown in the figure.



Let, initial charge on capacitor of capacitance C, when switch is closed, be Q_0, therefore, initial potential difference across it will be $V_0 = \frac{Q_0}{C}$.

Let, at any instant t charge on the capacitor is Q_t and therefore potential difference across it is $V_t = \frac{Q_t}{C}$. Accordingly, at that instant current through discharge is resistance R applying Kirchhoff's Loop Law in the circuit we have $V_t - i_tR = 0 \Rightarrow i_t = \frac{V_t}{R}$. During discharge charge on capacitor decreases, unlike charging, leads to $i_t = -\frac{d}{dt}Q_t \Rightarrow \frac{d}{dt}Q_t = -\frac{\frac{Q_t}{R}}{C} \Rightarrow \frac{dQ_t}{Q_t} = -\frac{dt}{RC}$. Integrating both sides we get $\int \frac{1}{Q_t}dQ_t = -\frac{1}{rC}\int dt \Rightarrow \ln Q_t = -\frac{t}{rC} + K \Rightarrow Q_t = e^{-\left(\frac{t}{rC}+K\right)} \Rightarrow Q_t = K'e^{-\frac{t}{rC}}$…(8). Here, both K and K' are integrating constants whose value depends upon initial condition $t = 0$, such that $K' = Q_0 = V_0C$ ...(9).

Accordingly, combining (8) and (9), $Q_t = V_0Ce^{-\frac{t}{RC}}$ …(10). Therefore, discharge currents through the resistor is $i_t = \frac{d}{dt}Q_t \Rightarrow i_t = \frac{E}{r}e^{-\frac{t}{rC_1}}$…(11).

Finally, during discharging of a capacitor, at any instant t , charge $\boldsymbol{Q_t}$ on it and current through the resistor $\boldsymbol{i_t}$ is-

(C) $\boldsymbol{Q_t = EC\left(1 - e^{-\left(\frac{t}{CR}\right)}\right)}$ (D) $\boldsymbol{i_t = \frac{E}{R}e^{-\left(\frac{t}{CR}\right)}}$

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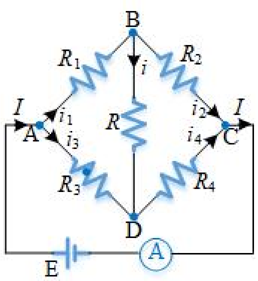
Appendix-II

Wheatstone Bridge

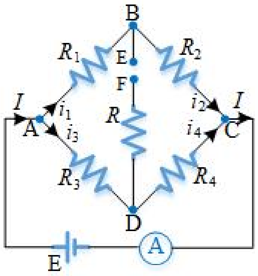
Wheatstone Bridge is a typical connection of resistors which bridges nodes B and D through a resistor, yet the branch B-D is an open circuit. It is a ***very important circuit*** *extensively used for different application in different names. It was developed by Samuel Hunter Christie (year 1833) and modified by Sir Charles Wheatstone year 1843) and known after the name of the latter.*



Typical connection of resistors is shown in the figure. The given circuit is redrawn, in a generic sense, with resistances that R_1, R_2, R_3, R_4 and R with branch currents i_1, i_2, i_3, i_4 and i, and nodes A,B,C and D are identified for analysis. As per initial premise current $i = 0$…(1), through resistance R. With this this value of R is to be determined.

As per Ohm's Law, is $i = \frac{V_B - V_D}{R}$…(2), considering assigned direction to the current from node B to D. Combining (1) and (2), $\frac{V_B - V_D}{R} = 0$ is possible when - **(a)** Either $V_B - V_D = 0$ …(3), or **(b)** $R \rightarrow \infty$**.** Taking possibility (a) in (3), $V_B = V_D$ …(4). Accordingly, voltage equation for branches

AB, BC, AD and DC we have – $V_A - V_B = i_1R_1$…(5), $V_A - V_D = i_3R_3$…(6), $V_B - V_C = i_2R_2$…(7), and $V_D - V_C = i_4R_4$…(8).



And as per Kirchhoff's current law at node B $i_1 = i + i_2$; using (1) $i_1 = i_2$…(9), likewise, at node D $i_3 + i = i_4 \Rightarrow i_3 = i_4$…(10).

Using (5) and (6) , $V_B - V_D = i_3R_3 - i_1R_1 = 0 \Rightarrow \frac{i_1}{i_3} = \frac{R_3}{R_1}$…(11). Likewise, using (7) and (8), we have $V_B - V_D = i_2R_2 - i_4R_4 = 0 \Rightarrow \frac{i_2}{i_4} = \frac{R_4}{R_2}$ …(12).

Now combining (12) with (9) and (10), $\frac{i_1}{i_3} = \frac{R_4}{R_2}$…(13). Comparing (11) and (13) $\frac{R_3}{R_1} = \frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$. …(13).

It concludes that $\frac{R_3}{R_1} = \frac{R_4}{R_2} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$ is a result of the basic premise $i = 0$, or vice-versa. Thus, any value of resistance $0 \leq R, \infty$ will do. However, to verify that the circuit satisfies the premise a current sensitive device viz. galvanometer is used in series with resistor R.

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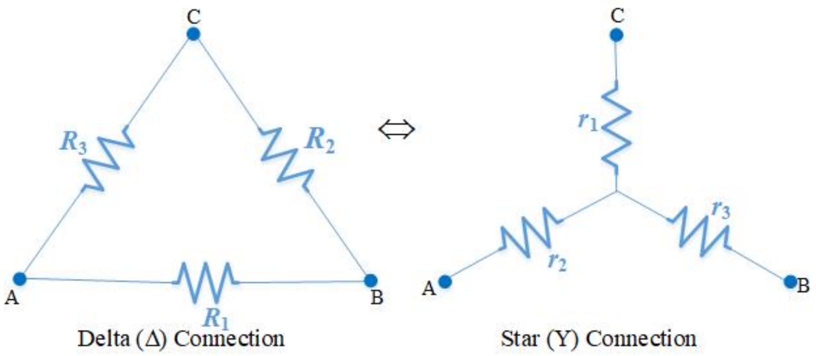
Appendix-III

Conversion of Delta-Star (Δ-Y) Equivalent of Resistance Network



Delta (Δ) Connection   Star (Y) Connection

Operation	Delta Connection	Star Connection	Eqn.
Equivalent resistance between nodes A-B	$R_{AB} = R_1 \| \| (R_2 + R_3)$ $\Rightarrow \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$	$R_{AB} = r_2 + r_3$	(1)
Equivalent resistance between nodes B-C	$R_{BC} = R_2 \| \| (R_1 + R_3)$ $\Rightarrow \frac{R_2(R_1+R_3)}{R_1+R_2+R_3}$	$R_{BC} = r_3 + r_1$	(2)
Equivalent resistance between nodes C-A	$R_{CA} = R_3 \| \| (R_2 + R_1)$ $\Rightarrow \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$	$R_{CA} = r_3 + r_2$	(3)

Delta to Star Conversion			
(2)-(1): $R_{AB} - R_{BC}$	$\frac{R_1(R_2+R_3)}{R_1+R_2+R_3} - \frac{R_2(R_1+R_3)}{R_1+R_2+R_3} \Rightarrow \frac{R_3(R_1-R_2)}{R_1+R_2+R_3}$	$r_1 - r_3$	(4)
(3)+(4): $R_{CA} + (R_{AB} - R_{BC})$	$\frac{R_3(R_1+R_2)}{R_1+R_2+R_3} + \frac{R_3(R_2-R_1)}{R_1+R_2+R_3} \Rightarrow \frac{2R_2R_3}{R_1+R_2+R_3}$	$2r_1$	(5)
Final Form of Conversion Formula	$\boldsymbol{r_1} = \frac{\boldsymbol{R_2R_3}}{\boldsymbol{R_1+R_2+R_3}}$; Likewise, $\boldsymbol{r_2} = \frac{\boldsymbol{R_1R_3}}{\boldsymbol{R_1+R_2+R_3}}$; $\boldsymbol{r_3} = \frac{\boldsymbol{R_1R_2}}{\boldsymbol{R_1+R_2+R_3}}$		(6)
Star to Delta Conversion			
Using (6)	$r_1r_2 + r_2r_3 + r_3r_1 = \left(\frac{R_2R_3}{R_1+R_2+R_3}\right)\left(\frac{R_1R_3}{R_1+R_2+R_3}\right) + \left(\frac{R_1R_3}{R_1+R_2+R_3}\right)\left(\frac{R_1R_2}{R_1+R_2+R_3}\right) + \left(\frac{R_1R_2}{R_1+R_2+R_3}\right)\left(\frac{R_2R_3}{R_1+R_2+R_3}\right) \Rightarrow \frac{R_1R_2R_3}{R_1+R_2+R_3}$		(7)
(7)/(6)	$\frac{r_1r_2+r_2r_3+r_3r_1}{r_1} = \frac{\frac{R_1R_2R_3}{R_1+R_2+R_3}}{\frac{R_2R_3}{R_1+R_2+R_3}}$		(8)
Final Form of Conversion Formula	$\boldsymbol{R_1} = \frac{\boldsymbol{r_1r_2+r_2r_3+r_3r_1}}{\boldsymbol{r_1}}$, likewise, $\boldsymbol{R_2} = \frac{\boldsymbol{r_1r_2+r_2r_3+r_3r_1}}{\boldsymbol{r_2}}$ and $R_3 = \frac{r_1r_2+r_2r_3+r_3r_1}{r_3}$		(9)

N.B.: 1. Derivation of $(Y - \Delta)$ is based on series-parallel equivalents resistances with algebraic manipulations.

2. A close observation of formulae derived, in relation to the figure, will help to develop an easy way to remember it.

3. Care in naming resistances in star connection and relating it to delta connection is important for ease of application.

4. A similar, but with a difference formulation for star-delta network of capacitances has been developed and is available in question bank on capacitors.

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