LET US DO SOME PROBLEMS-XXX

Some Problems from JEE MAIN 2021

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Q1. Let a, $b \in R$. If the mirror image of the point P(a, 6, 9)with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is (20, b, -1-9), then $|a+b|$is equal to:

(a)86 (b)88 (c)84 (d)90

Ans(b)

Q2. Let f be a twice differentiable function defined on R such that $f(0)=1$, $f'(0)=2$ and $f'(x)\neq 0$ for all x∈R . If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all x∈R, then the value of $f(1)$ lies in the interval

(a)(9,12) (b)(6,9) (c)(3,6) (d)(0,3)

Ans (b)

Q3. A possible value of $tan\left(\frac{1}{4}sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(a)$\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{7}}$ (c)$\sqrt{7}-1$ (d)$2\sqrt{2}-1$

Ans(b)

Q4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(a)$\frac{65}{2^7}$ (b) $\frac{135}{2^9}$ (c) $\frac{65}{2^9}$ (d) $\frac{35}{2^7}$

Ans (b)

Q5. The vector equation of the plane passing through the intersection of the planes

$\vec{r}.\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right) = 1$ and $\vec{r}.\left(\hat{\imath}-2\hat{\jmath}\right) = -2$, and the point (1,0,2) is:

(a) $\vec{r}.\left(\hat{\imath}-7\hat{\jmath}+3\hat{k}\right) = \frac{7}{3}$ (b) $\vec{r}.\left(\hat{\imath}+\widehat{7\jmath}+3\hat{k}\right) = 7$

(c) $\vec{r}.\left(3\hat{\imath}+7\hat{\jmath}+3\hat{k}\right)$=7 (d) $\vec{r}.\left(\hat{\imath}+7\hat{\jmath}+3\hat{k}\right) = \frac{7}{3}$

Ans (b)

Q6. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the coordinates of P are:

(a)(-2,8) (b)(1,5) (c)(3,13) (d)(2,8)

Ans (d)

Q7. Let a,b,c be in arithmetic progression. Let the centroid of the triangle with vertices (a,c), $(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

(a) $\frac{71}{256}$ (b) $-\frac{69}{256}$ (c) $\frac{69}{256}$ (d) $-\frac{71}{256}$

Ans(d)

Q8. The value of the integral $\int_1^3 [x^2-2x-2]\,dx$, where [x] denotes the greatest integer less than or equal to x, is :

(a)-4 (b)-5 (c) $-\sqrt{2}-\sqrt{3}-1$ (d) $-\sqrt{2}-\sqrt{3}+1$

Ans(c)

Q9. Let $f:R\rightarrow R$ be defined as

$$f(x)=\begin{cases} -55x, if\ x<-5 \\ 2x^3-3x^2-120x, if\ -5\le x\le 4 \\ 2x^3-3x^2-36x-336, if\ x>4 \end{cases}$$

Let A={x∈R : f is increasing}. Then A is equal to:

(a)(-5,-4)∪ (4,∞) (b)(-5,∞) (c)(-∞,-5) ∪ (4,∞) (d)(-∞,-5) ∪ (-4,∞)

Ans(a)

Q10. If the curve $y=ax^2+bx+c$, x∈R passes through the point (1,2) and the tangent line to this curve at origin is $y=x$, then the possible values of a,b,c are:

(a)$a=1, b=1, c=0$ (b)$a=-1, b=1, c=1$

(c)$a=1. b=0, c=1$ (d)$a=\frac{1}{2}, b=\frac{1}{2}, c=1$

Ans(a)

Q11. The negation of the statement ~p∧(p∨ q) is:

(a) ~p∧ q (b) p∧ ~q (c) ~p∨q (d) p∨~q

Ans(d)

Q12. For the system of linear equations: $x-2y=1$, $x-y+kz=-2$, $ky+4z=6$, k∈ R.

Consider the following statements:

(A)the system has unique solution if k≠ 2, k≠ -2

(B)the system has unique solution if k=-2

(C) the system has unique solution if k=2
(D) the system has no solution if k=2
(E) the system has infinite number of solutions if k$\neq$ -2

Which of the following statements are correct?
(a)B and E only (b)C and D only (c)A and D only (d) A and E only

Ans(c)

Q13. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?
(a) $x^2+9y^2=9$ (b) $2x^2-8y^2=9$ (c) $y^2=\frac{1}{6\sqrt{3}}x$ (d) $x^2+y^2=7$

Ans(a)

Q14. The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30^0. If the jet plane is flying at a constant height, then its height is:
(a) $1200\sqrt{3}m$ (b) $1800\sqrt{3}m$ (c) $3600\sqrt{3}m$ (d) $2400\sqrt{3}m$

Ans(a)

Q15. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2-B^2A^2)X=O$, where X is 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has:
(a)a unique solution (b)exactly two solutions
(c)infinitely many solutions (d)no solution

Ans(c)

Q16. If n≥ 2 is a positive integer, then the sum of the series ${}^{n+1}C_2+2({}^2C_2+{}^3C_2+{}^4C_2+\ldots+{}^nC_2)$ is:
(a) $\frac{n(n+2)(n+1)^2}{12}$ (b) $\frac{n(n-1)(2n+1)}{6}$
(c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\frac{n(2n+1)(3n+1)}{6}$

Ans(c)

Q17. If a curve $y=f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$, then for what value of b, $\int_1^2 f(x)dx=\frac{62}{5}$?
(a)5 (b) $\frac{62}{5}$ (c) $\frac{31}{5}$ (d)10

Ans(d)

Q18. The area of the region: R={(x, y) : $5x^2 \leq y \leq 2x^2+9$} is:

(a)$9\sqrt{3}$ (b) $12\sqrt{3}$ (c) $11\sqrt{3}$ (d) $6\sqrt{3}$

Ans (b)

Q19. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is

(a)2 (b)3 (c)4 (d)0

Ans (a)

Q20. If $a+\alpha=1$, $b+\beta=2$ and $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}, x \neq 0$, then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is

(a)2 (b)3 (c)4 (d)0

Ans(a)

Q21. Let $= \sqrt{-1}$. If $\frac{\left(-1+i\sqrt{3}\right)^{21}}{(1-i)^{24}} + \frac{\left(1+i\sqrt{3}\right)^{21}}{(1+i)^{24}} = k$ and $n = [|k|]$ be the greatest integral part of $|k|$. Then $\sum_{j=0}^{n+5}(j+5)^2 - \sum_{j=0}^{n+5}(j+5)$ is equal to

(a)310 (b)301 (c)103 (d)130

Ans(a)