Wave and Motion : Geometrical Optics & Dispersion -Typical Problems (Set 4)

*These question banks have been developed for students who are - **(a)** in initial stages of solving problems from text book or reference book so as to gain proficiency in application of concepts learnt, and **(b)** deprived of adequate exposure at learning. Such unprivileged and deprived students need guidance for stepwise application of concepts and associated mathematics, while evolving solutions. Here, main purpose is to inculcate in students an ability to appreciate physics and related mathematics involved in problems and apply them to reach a solution. Accordingly, illustrations have been made explanatory to the extent possible. Once, students get equipped with that capability, gradually they would be able to evolve optimized solutions and shortcuts suiting individually. Greater is the practice more intuitive becomes the optimization of steps. Those students who are at a stage of refining their problem solving skills or more apt at concepts may choose to skip illustrations or use them selectively to the best of their advantage.*

Science is a subject not to learn but a matter of realization through experiments and its visualization in surrounding. But, our target students are not equipped either to conduct experiment or an environment which facilitates them visualization of science and play around with it. This is where simulation is a technique to validate concepts and study effect of variation in parameters related to the concept. Education creates an opportunity of systematic learning concepts without reinventing the wheel. It is more apt in science education.

Solving typical problems with gradual increase in complexity helps to build power of visualization of concepts, without loss of confidence in one's ability. It requires reasonable proficiency in language to understand problem, in first go. Next comes evolving solution or answer based on concepts learnt. At this stage extremely simpler calculations are being skipped, with a hope that reader would be able to decipher intermediate steps.

Questions and problems appearing in competitive examinations are seldom encountered in real life, and are never straight application of formula. They demand integration of interdisciplinary knowledge. Yet ability to solve such typical problems, enhances competence to handle unknown problems speedily, correctly and with a greater degree of clarity and confidence, an essential attribute of thought process needed for success in life.

Mentors' Manual is one of the dimensions of the Gyan Vigyan Sarita through which efforts are being made to reach out to remote teachers through our experience of mentoring unprivileged children who severely lack in exposure. These students are disconnected from us for multiple reasons. Despite, efforts to establish direct interaction through Interactive Online Mentoring Sessions (IOMS) its reach to target students is extremely feeble. Yet, the IOMS has established as a working model of selfless mentoring of unprivileged children. This experience is being disseminated to the teachers spread out by writing of chapters of an open source Mentors' Manual.

India, growing digital, provides optimism to every student to be able to have an access to virtual laboratory; it is an alternative to physical laboratory. It provides an opportunity to carry out virtual experiments in an e-environment. In this environment excellent simulation videos are available on the web either free or on price. But, problem mostly encountered by students is in sequencing and scaling of concepts and selection of an appropriate video out of a big list available in web-search. This is severely distracting. Mentors are, however, the best persons to use these videos either to modulate and upgrade their illustrations or advise students a sequential list for each topic. Yet it does not rule out importance of hands-on by students in problem solving; it is called dry-run of concepts in the parlance of computer programming.

In light of this, this Question Banks includes problems from various sources and support them with illustrations. These illustrations are not just solutions but an attempt to bring home use of basics involved in solving problems. In this effort repository of problems from good books viz. Prof. H.C. Verma and a team of authors Robert Resnick, David Halliday and Kenneth S. Krane and many more have been used. These authors have graded questions while incorporating all concepts covered in the book. Thus it necessitates a student to read each chapter carefully before taking up questions.

In this stream of efforts Question Bank, Part-3, Set-4 with illustrations on Geometrical Optics has been uploaded on the web. Out of this few selected question are brought out here.

This initiative is of a small group of passionate persons who are focused to mentor unprivileged children with a sense of **Personal Social Responsibility (PSR)** in a *non-organizational, non-remunerative, non-commercial and non-political manner*. You are welcome to add value to this initiative by way of suggestion, advising correction or new type of questions, or any other form that suits to your passion and convenience.

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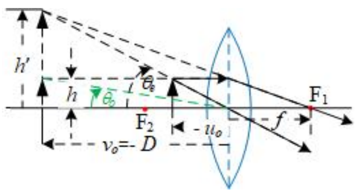
Wave and Motion: Optical Instruments & Dispersion– Typical Questions (With Answers and Illustration)

Q-1 The magnifying power of a converging lens used as a simple microscope is $\left(1+\frac{D}{f}\right)$. A compound microscope is a combination of two such converging lenses. Why don't we have magnifying $\left(1+\frac{D}{f_o}\right)\left(1+\frac{D}{f_e}\right)$? In other words, why can the object piece not be treated as a simple microscope but the eyepiece can?

A-1 Object lens does not act as simple microscope, and hence it is not a case of cascaded two simple microscopes.

I-1 Answering this question requires to revisit the principle of simple microscope and compound microscope; it is being illustrated here.

Simple Microscope: Ray diagram depicts object-lens, object of height h and its image of height h' at near distance D. In simple microscope object is so placed near the convex lens that its angle of vision θ_e coincides with the image at near distance D. Accordingly, $\theta_e=\frac{h}{u_0}=\frac{h\prime}{D}$ …(1). Further, for lenses, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{-D}-\frac{1}{-u_0}=\frac{1}{f}\Rightarrow\frac{1}{u_0}=\frac{1}{f}+\frac{1}{D}$. On multiplying h to the equation, $\frac{h}{u_0}=\frac{h}{f}+\frac{h}{D}$…(2). But, angular magnification is angle of vision of object and images placed at same distance and thus $m=\frac{\theta_e}{\theta_0}=\frac{\frac{h}{u_O}}{\frac{h}{D}}$; combining this with (1) and (2), $m=\frac{\frac{h}{f}+\frac{h}{D}}{\frac{h}{D}}=\frac{D}{f}+1$ …(3). Here f is the focal length of the simple microscope where $D\gg f\Rightarrow\frac{D}{f}\gg 1$ and thus $m\approx\frac{D}{f}$…(4) It is to be seen that in this case virtual image is formed by the lens with object and image on same side of the lens.



Compound Microscope: Compound microscope is a combination of two lenses object lens (O) having focal length f_O and eye piece (E) with focal length f_e. The latter is not called eye lens, should it not be confused lens of eye. The object lens form real and inverted image on the opposite sides of the lenses, while eye piece acts like simple microscope. Thus-two stage image formation is analyzed below –

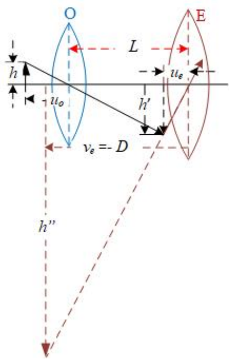


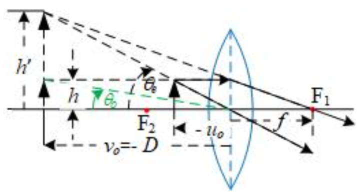
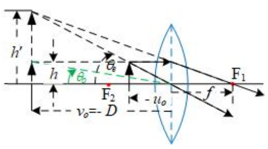
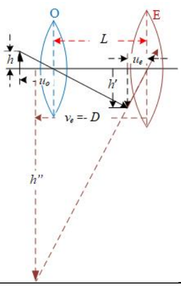
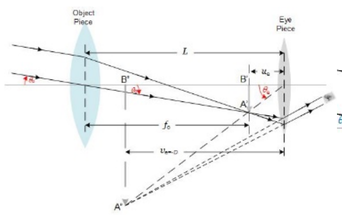
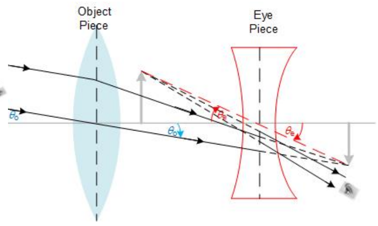
Image formation by O – As per lens formula, $\frac{1}{v_O}-\frac{1}{-u_0}=\frac{1}{f_O}\Rightarrow\frac{v_O}{u_0}=\frac{v_O}{f_O}-1$. Magnifying power of O is $m_O=\frac{h\prime}{h}=-\frac{v_O}{u_0}=-\left(\frac{v_O}{f_O}-1\right)\Rightarrow m_O=1-\frac{v_O}{f_O}$…(5).

Image formation by E – Since, it acts like simple microscope and hence utilizing (3) we have, $m_e=1+\frac{D}{f_e}$.. (6). Hence, net amplification of compound microscope is $m=m_O\times m_e=\left(1-\frac{v_O}{f_O}\right)\times\left(1+\frac{D}{f_e}\right)$. It is to be noted that in compound microscope is so designed that image formed by O is just infront of the E and $L\approx v_O$. ***Accordingly the magnification is*** $\boldsymbol{m=\left(1-\frac{L}{f_O}\right)\times\left(1+\frac{D}{f_e}\right)}$***, and not taking to two lenses as cascaded two simple microscopes.*** Further, approximation $L\gg f_O\Rightarrow\frac{L}{f_O}\gg 1$ and hence $m\approx\left(-\frac{L}{f_O}\right)\times\left(1+\frac{D}{f_e}\right)$.

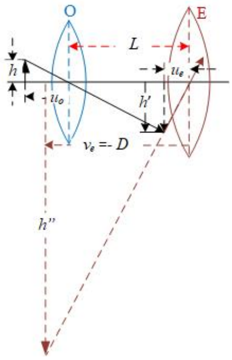
Q-2 An object is placed at a distance of 30 cm from a converging lens of focal length 15 cm. A normal eye (near point 25 cm, far point infinity) is placed close to the lens on the other side.

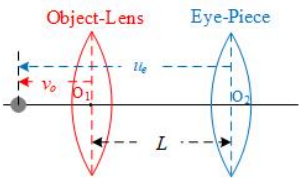
(a) Can the eye see the object clearly?
(b) What should be the minimum separation between the lens and the eye so that eye can clearly see the object?
(c) Can a diverging lens, placed in contact with the converging lens, help in seeing the object clearly when the eye is close to the lens?

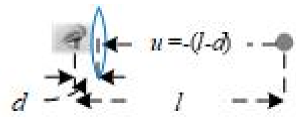
A-2 No, 55 cm, Yes if focal length of diverging lens is nearly 9.4 cm.

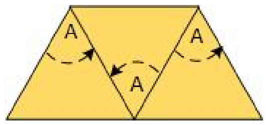
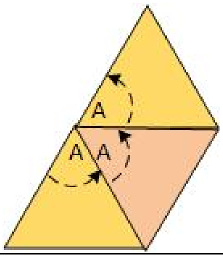
I-2	Each part is elaborated separately – **Part (a):** As per lens formula , $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, here using Cartesian sign convention, $u=-30$ cm, $f=15$ cm and hence position of image v is $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}\Rightarrow\frac{1}{v}=\frac{1}{15}+\frac{1}{-30}\Rightarrow\frac{1}{v}=\frac{1}{30}$. Since, eye is close to lens and hence image will be formed behind the eye, which is **not** clearly visible. **Part (b):** For clarity of vision, object or its image should be placed at near distance $D=25$ cm from the eye. Therefore, distance of eye from the lens should be equal to $v+D=30+25=\mathbf{55\ cm}$. **Part (c):** Diverging lens if placed in contact with converging lens of focal length f_1 will form an intermediate virtual image on the same side of the lens where object is such that $\frac{1}{v'}=\frac{1}{f_1}+\frac{1}{u}$. This virtual image will act as object for convex lens to form a final image such that $\frac{1}{v}-\frac{1}{v'}=\frac{1}{f_2}$. Accordingly, $\frac{1}{v}=\frac{1}{f_2}+\frac{1}{v'}\Rightarrow\frac{1}{v}=\frac{1}{f_2}+\left(\frac{1}{f_1}+\frac{1}{u}\right)\Rightarrow\frac{1}{v}-\frac{1}{u}=\frac{1}{f_2}+\frac{1}{f_1}$. Therefore, for object to be visible clearly it should be away from eye at $v=D=-25$ cm. Thus, it requires focal length of the diverging lens f_2 to be $\frac{1}{f_2}=\frac{1}{v}-\frac{1}{u}-\frac{1}{f_1}\Rightarrow\frac{1}{f_2}=\frac{1}{-25}-\frac{1}{-30}-\frac{1}{15}\Rightarrow\frac{1}{f_2}=\frac{1}{30}-\frac{1}{25}-\frac{1}{15}$ or $f_2=-\frac{75}{8}\approx-9.4$ cm
Q-3	An object is placed at a distance u from a simple microscope of focal length f. The angular magnification obtained depends (a) On f but not on u (b) On u but not on f (c) On f as well as u (d) Neither on f as well as u
A-3	(c)
I-3	 Ray diagram of simple microscope depicts object-lens, object of height h and its image of height h' at near distance D. In simple microscope object is so placed near the convex lens that its angle of vision θ_e coincides with the image at near distance D. Accordingly, $\theta_e=\frac{h}{u}=\frac{h'}{D}$ …(1). Here, in figure u_0 is represented in expression as u. Further, for lenses, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{-D}-\frac{1}{-u}=\frac{1}{f}\Rightarrow\frac{1}{u}=\frac{1}{f}+\frac{1}{D}$. On multiplying h to the equation, $\frac{h}{u}=\frac{h}{f}+\frac{h}{D}$…(2). But, angular magnification is ratio angle of vision of image and object placed at same distance and thus $m=\frac{\theta_e}{\theta_0}=\frac{\frac{h}{u}}{\frac{h}{D}}=\frac{D}{u}$. For maximum amplification $f\ll$ and $u\to f$ while, near distance D is constant. Hence, angular amplification depends upon f as well as u, as provided in option (c). Hence, **answer is option (c).**
Q-4	In which of the following the final image is erect? (a) Simple microscope (b) Compound microscope (c) Astronomical telescope (d) Galilean telescope
A-4	(a), (d)
I-4	Geometrical optics is shown below in the same sequence – Simple Microscope (Erect Image)  Compound Microscope (Inverted Image)  Astronomical Telescope (Inverted Image)  Galilean Telescope (Erect Image) 

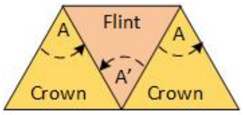
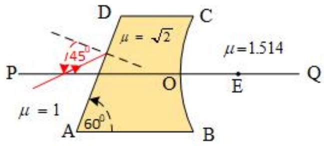
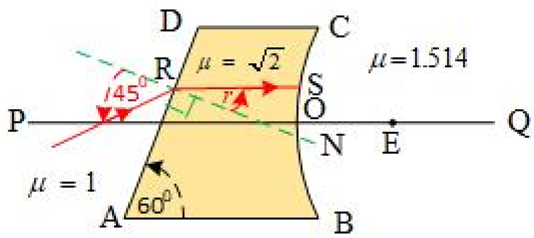
	Thus seeing the diagrams, **answer is option (a) and (d).**
Q-5	Mark the correct options (a) If the far point goes ahead, the power of the divergent lens should be reduced (b) If near point goes ahead, the power of the convergent lens should be reduced (c) If far point is 1m away from the eye, divergent lens should be used (d) If near point is 1m away from the eye, divergent lens should be used
A-5	(a), (c)
I-5	Each of the options will have to be analyzed, using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ –, to determine the correct options – **Option (a):** Given data is $u=-\infty$, $v=-z$ the far point, then $\frac{1}{-z}-\frac{1}{-\infty}=\frac{1}{f}\Rightarrow f=-z$. Here (-) ve value of focal length of lens implies that it is divergent lens. Now, if far point goes ahead to $-x$ i.e magnitude of it moves towards to lens and hence $\|-x\|<\|-z\|$. Since power of lens is $P=\frac{1}{f}$ and, therefore, with this inverse proportionality, $\frac{1}{\|-x\|}>\frac{1}{\|-z\|}\Rightarrow\frac{1}{-x}<\frac{1}{-z}\Rightarrow P_x<P_z$. Accordingly, despite reduction in magnitude of focal length power of lens is reduced, therefore, **option (a) is correct**. **Option (b):** Given data is $u=-D$ the near point and the image is also at the near point $v=-D$. Therefore, the lens formula takes the form $\frac{1}{-D}-\frac{1}{-D}=\frac{1}{f}\Rightarrow f=\infty\Rightarrow P=\frac{1}{\infty}=0$, i.e.no lens is required. Now, if near point goes ahead to $u=-x$ i.e magnitude of it moves towards to lens and hence $\frac{1}{-D}-\frac{1}{-x}=\frac{1}{f}\Rightarrow\frac{1}{f}=f=\frac{1}{x}-\frac{1}{D}$. Since, $\|-x\|<\|-D\|\Rightarrow\frac{1}{\|-x\|}>\frac{1}{\|-D\|}\Rightarrow\frac{1}{x}>\frac{1}{D}$, focal length is (+) i.e. it is convergent lens, therefore, $P_x>P_D$, which contradicts the given statement. Hence, **option (b) is incorrect**. **Option (c):** Given data is $u=-\infty$, $z=v=-1$ m, then, $\frac{1}{f}=\frac{1}{-1}-\frac{1}{-\infty}\Rightarrow\frac{1}{f}=-1\Rightarrow f=-1$ m. The (-)ve sign of the focal length confirms that the required lens id divergent and asserts statement in option (c), hence option (c) is correct. **Option (d):** Given that near point is $v=-1$, m while required value is $u=D=-0.25$m, Hence, as per lens formula $\frac{1}{f}=\frac{1}{-1}-\frac{1}{-0.25}\Rightarrow\frac{1}{f}=3\Rightarrow f=\frac{1}{3}$, the positive value of focal length of lens confirms that the lens is convergent, and it contradict statement in option (d). Hence, **option (d) is incorrect.** Hence, **answer is options (a) and (c).**
Q-6	A simple microscope has a magnifying power of 3.0 when the image is formed at the near point (25 cm) of a normal eye. (a) What is the focal length? (b) What will be its magnifying power if the image is formed at infinity?
A-6	(a) 12.5 cm (b) 2.0
I-6	Magnifying power of a simple microscope is $m=1+\frac{D}{f}$, using the given data, $3.0=1+\frac{25}{f}\Rightarrow f=\frac{25}{2.0}=$ **12.5 cm, is answer of part (a).** When image is formed at infinity then in the magnification formula of simple microscope $\frac{D}{f}\gg$ and thus magnifying power approximates to $m=\frac{D}{f}\Rightarrow m=\frac{25}{12.5}=\mathbf{2}$ **, is answer of part (b).** **Thus answers are (a) 12.5 cm (b) 2.0.**
Q-7	Find the maximum magnifying power of a compound microscope having a 25 diopter lens as a object piece, a 5 diopter lens as the eyepiece and a separation 30 cm between the two lenses. The least distance for clear vision is 25 cm.
A-7	8.4

I-7	Compound microscope is a combination of two lenses object lens (O) having focal length f_o and eye piece (E) with focal length f_e. The latter is not called eye lens, should it not be confused lens of eye. The object lens form real and inverted image on the opposite sides of the lenses, while eye piece acts like simple microscope. Thus-two stage image formation is analyzed below – ***Image formation by O*** – As per lens formula, $\frac{1}{v_o} - \frac{1}{-u_0} = \frac{1}{f_O} \Rightarrow \frac{v_o}{u_0} = \frac{v_o}{f_O} - 1$. Magnifying power of O is $m_o = \frac{h\prime}{h} = -\frac{v_o}{u_0} = -\left(\frac{v_o}{f_O} - 1\right) \Rightarrow m_o = 1 - \frac{v_o}{f_O}$. ***Image formation by E*** – Since, it acts like simple microscope and hence utilizing (3) we have, $m_e = 1 + \frac{D}{fe}$.. (6). Hence, net amplification of compound microscope is $m = m_o \times m_e = \left(1 - \frac{v_o}{f_O}\right) \times \left(1 + \frac{D}{fe}\right)$. In com pound microscope $f_o \approx u_o$ and $\frac{v_o}{u_o} \gg 1$, therefore, it approximates to $m = -\frac{v_o}{u_o} \times \left(1 + \frac{D}{fe}\right)$. ..(1) Now, solving the problem for stage-wise amplifications, starting from eye piece, we have focal length of object lens is $f_o = \frac{1}{m_0} = \frac{1}{25} = 4$ cm and of eye-piece $f_e = \frac{1}{m_e} = \frac{1}{5} = 20$ cm. For clear vision through eye piece $D = v_e = -25$ cm and using lens formula, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{fe} \Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{fe} \Rightarrow u_e = \frac{v_e f_e}{fe - v_e}$. It solves into $u_e = \frac{20\times(-25)}{20-(-25)} = -\frac{100}{9}$ cm. Given that $L = 30 = v_o + u_e \Rightarrow v_o = 30 - \frac{100}{9} = \frac{170}{9} = 18.89$ cm. Therefore, for object lens $\frac{1}{v_o} - \frac{1}{u_0} = \frac{1}{f_O} \Rightarrow u_0 = \frac{v_o f_O}{f_O - v_O} = \frac{\frac{170}{9}\times 4}{4 - \frac{170}{9}} = \frac{170\times 4}{36-170} = -5.07$cm. Using the available data in (1) $m = \frac{18.89}{-5.07} \times \left(1 + \frac{25}{20}\right) = -8.4$. Hence, 8.4 is the answer.  **N.B: (1)** In final amplification, sign is insignificant. **(2)** Answer is using principle of SDs. **(3)** Form of formula to be used in a problem depends upon available data and the best way to reach the required answer or result; an essential consideration in each problem.
Q-8	An eye can distinguish between two points of an object if they are separated by more than 0.22 mm when object is placed at 25 cm from the eye. The object is now seen by a compound microscope having 20 D object-piece and 10D eyepiece separated by 20 cm. The final image is formed at 25 cm from the eye. What is the minimum separation between two points of the object which can be distinguished?
A-8	0.04 mm
I-8	Let m is the magnifying power of the lens then for minimum separation d to be visible under microscope necessary condition is $m = \frac{d'}{d}$, here $d' = 0.22$ mm is the minimum separation visible with bare eye when placed at near distance $D = 25$ cm. Accordingly, $d = \frac{d'}{m}$…(1) Thus problem requires to determine m wfrom the available data $L = 20$ cm, $P_o = \frac{1}{f_O} = 20 \Rightarrow f_O = \frac{1}{20} = 0.05$m =5 cm, $P_e = \frac{1}{fe} = 10 \Rightarrow f_e = \frac{1}{10} =$ 0.10 m = 10 cm. We know that, in compound microscope, $m = \frac{v_o}{u_o} \times \left(1 + \frac{D}{fe}\right)$…(2). Therefore, unknown are v_o and u_o. Which are determined from the available data starting from eye piece, and is illustrated below – Position of real image infront of eye piece, give that $u_e = D = 25$ cm, as per lens formula , $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{fe}$ is $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{fe} \Rightarrow u_e = \frac{v_e \times f_e}{fe - v_e} \Rightarrow u_e = \frac{(-25)\times 10}{10-(-25)} = -\frac{50}{7}$ cm. Since, in the instrument $L = v_o + \|u_e\| \Rightarrow v_o = 20 - \frac{50}{7} = \frac{90}{7}$ cm. With this, $\|u_o\| = \left\|\frac{v_o \times f_O}{f_O - v_o}\right\| = \left\|\frac{\frac{90}{7}\times 5}{5 - \frac{90}{7}}\right\| = \frac{450}{55} = \frac{90}{11}$ cm. Therefore, using (2) magnifying power of lens is $m = \frac{\frac{90}{7}}{\frac{90}{11}} \times \left(1 + \frac{25}{10}\right) = \frac{11}{7} \times 3.5 = 5.5$. Therefore, using (1) we have $d = \frac{0.22}{5.5} = \frac{0.2}{0.5} = \mathbf{0.04}$ **mm is the answer**. **N.B.:** Generally, data given in problems is such that calculations can be minimized by using fractional values till end, at this level aim is to check conceptual clarity, and not numerical ability. Any haste in

	calculating intermediate values in decimal form makes calculations lengthy, more time consuming and hence prone to error. This is a good example.
Q-9	A compound microscope consists of an object-piece of focal length 1 cm and an eye piece of focal length 5 cm. An object is placed at a distance of 0.5 cm from the object piece. What should be separation between the lenses so that the microscope projects an inverted real image of the object on a screen 30m behind the eyepiece?
A-9	5 cm
I-9	It is given that compound microscope forms an inverted image on a screen behind eye-piece, it implies that the final image is real and since it is inverted at $v_e = +30$ cm, and separation between the two lenses $L = v_o + u_e$.  Using lens formula for eye piece, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$. Using the available data $\frac{1}{u_e} = \frac{1}{30} - \frac{1}{5} \Rightarrow u_e = \frac{30\times5}{5-30} = -6$ cm. Further, with the given data for object-lens $\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \Rightarrow \frac{1}{v_o} = \frac{1}{u_o} + \frac{1}{f_o} \Rightarrow \frac{1}{v_o} = \frac{1}{-0.5} + \frac{1}{1} \Rightarrow \frac{1}{v_o} = 1$ or $v_o = -1$. This geometry leads to a situation as shown in the figure where $L = \lvert u_e \rvert - \lvert v_o \rvert = 6 - 1$ or length of tube $L =$ **5 cm is the answer.** **N.B.:** It is a good example of formation of a real image from virtual image.
Q-10	An optical instrument used for angular magnification has a 25 D object-piece and a 20 D eyepiece. The tube length is 25 cm when the eye is least strained. (a) Whether it is microscope or a telescope? (b) What is the angular magnification produced?
A-10	Microscope, 33
I-10	Each part is being elaborated separately – **Part (a):** In microscope $f_e > f_o$ while in telescope $f_e < f_o$. With five data for object-lens $f_o = \frac{1}{P_o} = \frac{1}{25}$ m or $f_o = 4$ cm, and for eye-piece $f_e = \frac{1}{P_e} = \frac{1}{20}$ m or $f_e = 5$ cm. This goes in with instrument being **microscope, is the answer**. **Part (b):** Angular magnification of microscope is $m = \frac{v_o}{u_o} \times \left(1 + \frac{D}{f_e}\right)$…(1). with known data next aim is to determine u_o and v_o with the available data including $v_e = D = -25$ cm. **Step 1:** Using lens formula for eye-piece $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{D} - \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{-25} - \frac{1}{5} \Rightarrow u_e = -\frac{25}{6} = -4.2$ cm. Further, from geometry of microscope $L = v_o + \lvert u_e \rvert$. Therefore, $v_o = L - \lvert u_e \rvert \Rightarrow v_o = 25 - 4.2 = 21.8$ cm. **Step 2:** Using lens formula for object-lens $\frac{1}{v_o} - \frac{1}{u_0} = \frac{1}{f_o} \Rightarrow \frac{1}{u_0} = \frac{1}{v_o} - \frac{1}{f_o} \Rightarrow u_0 = \frac{v_o \times f_o}{f_o - v_o} \Rightarrow u_0 = \frac{21.8\times4}{4-21.8} = -4.9$cm **Step 3:** Using the available data $m = \left\lvert \frac{21.8}{4.0} \times \left(1 + \frac{25}{5}\right) \right\rvert \approx$ **33 is the answer**. **N.B:** Using the approximation formula $u_e = f_e \Rightarrow L = v_o + f_e$ and therefore $m = \frac{v_o}{u_o} \times \frac{D}{f_e}$, lead to a different result $m = 20$, having a wide departure from the answer arrived at above.
Q-11	A person wears glasses of power -2.5 D. Is the person farsighted or nearsighted? What is the far point of the person without glasses?
A-11	Nearsighted, 40 cm
I-11	Power of the lens given is (-)ve it implies that the lens is concave which diverging in nature and is needed to correct the image of a distant which is formed by the eye-lens near it rather than on retina. It is **the case of nearsightedness.** Further, $P = \frac{1}{f} = -2.5 \Rightarrow f = \frac{1}{P} \Rightarrow f = \frac{1}{-2.5} = -\mathbf{0.4}$ m= - 40 cm. As per the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-u} - \frac{1}{\infty} \Rightarrow u = \lvert -f \rvert = 40$ cm, is the answer/ **N.B.:** Far point and near points are expressed as unsigned values.

Q-12	A person has near point at 100 cm. What power of lens is needed to read at 20 cm if he uses (a) contact lens, (b) spectacles having glasses 2.0 cm separated from the eyes?
A-12	+4 D, +4.56 D
I-12	Given that near point i.e. $v = -100$ cm, while lens is needed to read at $u = -20$. Therefore, first focal length of contact lens needed to read the object, using lens formula is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-100} - \frac{1}{-20} \Rightarrow \frac{1}{f} = \frac{4}{100} \Rightarrow \frac{1}{f} = 0.04$ cm^{-1}. Therefore, powers of lens is $P = \frac{1}{f} \times 100 = +4$ D. When external lenses are separated by $d = 2$ cm, and $l = 20$ cm as shown in the figure. Then $u = -(20-2) = -18$ cm. Hence, again using lens formula, the focal length of the spectacles is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-100} - \frac{1}{-18} \Rightarrow \frac{1}{f} = \frac{41}{900}$. Therefore, powers of lens is $P = \frac{41}{900} \times 100 = +4.56$ D.  **N.B.:** While calculating power of lens, focal length is taken in meter and hence multiplier 100 is used.
Q-13	A lady cannot see objects closer than 40 cm from left eye and closer than 100 cm from right eye. While on a mountaineering trop, she is lost from her team. She tries to make an astronomical telescope from her reading glasses to look for her teammates. (a) Which glass should she use as the eyepiece? (b) What magnification can she get with relaxed eye?
A-13	(a) Right lens, (b) 2
I-13	Given data stipulates near distance of left eye $D_L = 0.4$ m and of right eye $D_R = 1$ m. *In astronomical telescope focal length of eye piece is less than focal length of object piece.* In which arrangement the lady uses her glasses requires determination of focal length of both the glasses. While reading text is placed at equal distance from both the eyes say $u_L = u_L = u - 0.25$ m, while from the given data $v_L = -D_L = -0.4$ m and $v_R = -D_R = -1$m. Using the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ for left glasses $\frac{1}{f_L} = \frac{1}{v_L} - \frac{1}{u_L} \Rightarrow \frac{1}{f_L} = \frac{1}{-0.4} - \frac{1}{-0.25} \Rightarrow \frac{1}{f_L} = 4 - 2.5 = 1.5 \Rightarrow \boldsymbol{f_L = 0.\dot{6}}$ **m**. Likewise, focal length of right glasses $\frac{1}{f_R} = \frac{1}{v_R} - \frac{1}{u_R} \Rightarrow \frac{1}{f_R} = \frac{1}{-1} - \frac{1}{-0.25} \Rightarrow \frac{1}{f_R} = 4 - 1 = 3 \Rightarrow \boldsymbol{f_R = 0.\dot{3}}$. From the derived focal length of the glasses, $f_R < f_L$, hence **Right glass shall be used as eye piece, is answer of part (a).** From answer of part (a), $f_e = f_R = 0.\dot{3}$ and $f_o = f_L = 0.\dot{6}$ and magnifying power of astronomical telescope is $m = \frac{f_o}{f_e} = \frac{0.\dot{6}}{0.\dot{3}} =$ **2, is the answer of part (b).** **Thus, answer is (a) Right lens, (b) 2.** **N.B.:** (a) It is convenient to convert all distances since in power of lens focal length lens is in meter. (b) Cartesian sign convention applies to parameters in lens formula and not to near or far distances.
Q-14	The angular dispersion produced by a prism – (a) increases if average refractive index increases (b) increases if the average refractive index decreases (c) remains constant whether the average refractive index increases or decreases (d) has no relation with the average refractive index
A-14	(a)
I-14	Angular dispersion depends upon angle of deviation for different wavelengths, which in turn depends on refractive index of the medium for the corresponding wavelength as per formula $\delta = (\mu - 1)A$, while refracting angle or also called angle of prism A remains constant. If average refractive index increases so also δ as per relationship given above and therefore $\Delta\delta = \delta_v - \delta_r$ also increases to increase angular dispersion.
Q-15	A prism can produce a minimum deviation δ in a light beam. If three prisms are combined, the minimum deviation that can be produced in this beam is – (a) 0 (b) δ (c) 2δ (d) 3δ
A-15	(b)

I-15	Given that there are three prisms having minimum deviation δ , It is required to determine angle of deviation of the combination of three prisms. Question does not specify orientation of the vertex of the prisms. Therefore, there are two possible combinations – **Combination I:** If two identical prisms are combined with their refracting angles inverted, then the second prism cancels both deviation and dispersion produced by the earlier one. While adding third identical prism nullifies earlier combination and acts like a prism creating both deviation and dispersion. Thus, net minimum deviation is $\boldsymbol{\delta}$ **as per option (b)**   **Combination II:** When all the prisms have same orientation of their vertices it causes cascading of both deviation by each prism and hence net deviation is 3δ as per option (d). Thus, **answer is option (b) and (d)**
Q-16	By properly combining two prisms made of different materials, it is possible to (a) Have dispersion without average deviation (b) Have deviation without dispersion (c) Have both dispersion and average deviation (d) Have neither dispersion nor average deviation
A-16	(a), (b), (c)
I-16	Average angle of deviation of a prism is for yellow colour which is nearly mean wavelength of the spectrum is $\delta = (\mu - 1)A$. Further, combination of prism implies that the two prisms are with inverted vertices and therefore net deviation would be $\delta_y = \left((\mu_y - 1)A\right) - \left((\mu'_y - 1)A'\right)$, where μ_y and μ'_y are refractive indices of the two materials of the two prisms, while A and A' angle of the two prisms respectively. It is possible to change angles of prisms to achieve $\delta_y = 0$, but with these angles of prisms δ_r and δ_v cannot be zero since μ is not linearly dependent on wave length. Hence while average deviation is zero, there would be dispersion. Thus, **option (a) is correct.** Extending the discussion in the above para while adjusting angle of the two inverted prisms to have equal deviation in angles of deviation of two prism $\Delta\delta = \delta_v - \delta_r$ and $\Delta\delta' = \delta'_v - \delta'_r$, the non-linearity of μ will not let $\delta_y - \delta'_y = 0$. Hence, dispersion free combination will have deviation. **Thus option (b) is correct**. In case there is no effort to match either mean deviation or dispersion of the two inverted prisms, it will have both deviation and dispersion. Thus, **option (c) is correct.** In light of the above discussions, it is not possible to make the combination deviation free and dispersion free. Thus option (d) is incorrect. Thus, **answer is options (a), (b) and (c).**
Q-17	A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of the flint and crown glasses for the mean ray are 1.620 and 1.518 respectively. If the refracting angles of the flint prism is 6^0, what should be the refracting angle of the crown prism?
A-17	7^0
I-17	Average angle of deviation of a prism is for yellow colour which is nearly mean wavelength of the spectrum is $\delta = (\mu - 1)A$. Further, combination of prism implies that the two prisms are with inverted vertices and therefore net deviation would be $\delta_y = \left((\mu_y - 1)A\right) - \left((\mu'_y - 1)A'\right)$, where μ_y and μ'_y are refractive indices of the two materials of the two prisms, while A and A' angle of the two prisms respectively. Therefore, for $\delta_y = 0$, essential condition is $\left((\mu_y - 1)A\right) = \left((\mu'_y - 1)A'\right)$. From the given data let for crown glass $\mu_y = 1.518$ and refracting angle A is to be determined, while for flint glass $\mu'_y = 1.620$ and refracting angle $A' = 6^0$. Therefore,$A = \frac{(\mu'_y - 1)A'}{(\mu_y - 1)} \Rightarrow A = \frac{(1.620-1)6}{(1.518-1)} = \frac{0.620\times 6}{0.518} = 7.2 \approx 7^0$. Thus, **answer based on principle of SDs is $\mathbf{7^0}$.**

Q-18	Three thin prisms are combined as shown in the figure. The refractive if the crown glass for red, yellow and violet rays are μ_r, μ_y and μ_v respectively and for the flint glass are μ'_r, μ'_y and μ'_v respectively. Find the ratio $\frac{A'}{A}$ for which (a) There is no net angular dispersion, (b) There is no net deviation in the yellow ray 
A-18	(a) $\frac{2(\mu_v-\mu_r)}{\mu'_v-\mu'_r}$ (b) $\frac{2(\mu_y-1)}{\mu'_y-1}$
I-18	Angular deviation of a thin prism is $\delta = (\mu - 1)A$. Let for a certain ray refractive index for crown glass is μ_c and for flint glass it is μ_c. Therefore for the combination as given, deviation produced by each of the two crown glass prisms $\delta_c = (\mu_c - 1)A$ would be cumulative while that of flint glass would be subtractive $\delta_f = (\mu_f - 1)A'$. Thus, net deviation for the ray would be $\Delta\delta = 2\delta_c - \delta_f = 2(\mu_c - 1)A - (\mu_f - 1)A'$. Accordingly, net angular deviation $\Delta\delta = 2(\mu_c - 1)A - (\mu_f - 1)A'$…(1) Angular dispersion for a system of prisms is $\omega = \frac{\mu_v-\mu_r}{\mu_y-1} \Rightarrow \omega = \frac{(\mu_v-1)A-(\mu_r-1)A}{(\mu_y-1)A} \Rightarrow \omega = \frac{\delta_v-\delta_r}{\delta_y-1}$. Extending this logic, for no net angular dispersion is $\Delta\delta_v - \Delta\delta_r = 0 \Rightarrow \Delta\delta_v = \Delta\delta_r$. Therefore, using (1) we have $2(\mu_r - 1)A - (\mu'_r - 1)A' = 2(\mu_v - 1)A - (\mu'_v - 1)A' \Rightarrow 2(\mu_r - \mu_v)A = (\mu'_r - \mu'_v)A' \Rightarrow \frac{A'}{A} = \frac{2(\mu_r-\mu_v)}{\mu'_r-\mu'_v}$ **is answer of part (a).** For no angular deviation in yellow ray, from (1) $\Delta\delta_y = 0 \Rightarrow 2(\mu_y - 1)A - (\mu'_y - 1)A' = 0 \Rightarrow \frac{A'}{A} = \frac{2(\mu_y-1)}{\mu'_y-1}$, **is answer of part (b).** Hence, **answer is (a)** $\boldsymbol{\frac{2(\mu_v-\mu_r)}{\mu'_v-\mu'_r}}$ **(b)** $\boldsymbol{\frac{2(\mu_y-1)}{\mu'_y-1}}$
Q-19	Figure shows an irregular block of material of refractive index $\sqrt{2}$. A lay of light strikes the face AB as shown in the figure. After refraction it is incident on spherical surface CD of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet PQ at E. Find distance OE upto two places of decimal. 
A-19	6.06 m
I-19	This is the case of double refraction of a ray passing through three different mediums having refractive indices $\mu_1 = 1$, $\mu_2 = \sqrt{2}$ and $\mu_3 = 1.514$ where first refraction is on a plane surface and it will abide Snell's law $\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$. With given data where $i = 45^0$, $\sin r = \frac{\sin i \times \mu_1}{\mu_2} \Rightarrow \sin r = \frac{\frac{1}{\sqrt{2}}\times 1}{\sqrt{2}} = \frac{1}{2} \Rightarrow \sin r = \sin 30^0 \Rightarrow r = 30^0$. This refracted ray RS, is geometrically parallel to the block base AB or parallel to the axis PQ of the block.  The second refraction from medium of refractive index μ_2 to μ_3 is through spherical surface of radius $R = 0.4$ m. and it will abide by $\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3-\mu_2}{R}$. Since, refracted ray ES is parallel to the axis of the block $u - \infty$. Thus using the available data and that $\sqrt{2} = 1.414$ we have $\frac{1.514}{v} - \frac{1.414}{\infty} = \frac{1.514-1.414}{0.4} \Rightarrow v = 1.514 \times 4 = \mathbf{6.06}$ **m, is the distance of E from O. is the answer**.