LET'S DO SOME PROBLEMS - XXXVIII

(FOCUS: SINGAPORE MATHEMATICAL OLYMPIADS)

Prof. SB Dhar

Some questions from the Singapore Mathematical Olympiad are selected here to understand the standard and mode of questions for those who are aspiring to appear in the Mathematical Olympiads.

QUESTIONS

Q1. If $a = 8^{53}, b = 16^{41}$ and $c = 64^{27}$, then which of the following inequalities is true?

(a)$a>b>c$ (b)$c>b>a$ (c)$b>a>c$
(d)$b>c>a$ (e)$c>a>b$

Ans. (d)

Q2. If *a*, *b*, *c* are real numbers such that $|a-b|=1, |b-c|=1, |c-a|=2$ and *abc = 60*, then find the value of $\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}-\frac{1}{a}-\frac{1}{b}+\frac{1}{c}$

(a) $\frac{1}{30}$ (b) $\frac{1}{20}$ (c) $\frac{1}{10}$
(d) $\frac{1}{4}$ (e)None of these

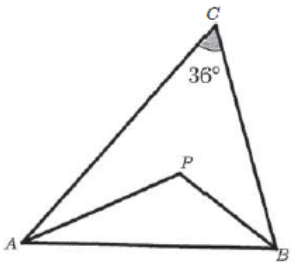
Ans.(b)

Q3. If *x* is complex number satisfying $x^2+x+1=0$, what is the value of $x^{49}+x^{50}+x^{51}+x^{52}+x^{53}$?

(a)-1 (b)-$\frac{1}{2}$ (c)0
(d) $\frac{1}{2}$ (e)1

Ans.(a)

Q4. In ΔACB, ∠ACB=36^0 and the interior angle bisectors of ∠CAB and ∠ABC intersect at P. Find ∠APB.



(a)72^0 (b)108^0 (c)126^0
(d)136^0 (e)None of these

Ans.(b)

Q5. Find the number of integer pairs *x,y* such that *xy-3x+5y=0*

(a)1 (b)2 (c)4
(d)8 (e)16

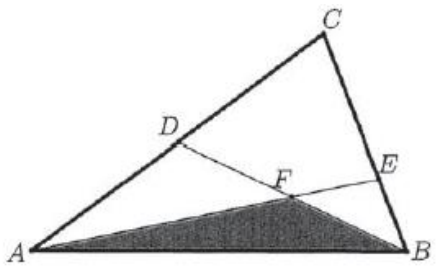
Ans.(d)

Q6. Five young ladies were seated around a circular table. Miss Ong was sitting between Miss Lim and Miss Mak. Ellie was sitting between Cindy and Miss Nai. Miss Lin was between Ellie and Amy. Lastly, Beatrice was seated with Miss Poh on her left and Miss Mak on her right. What is Daisy's surname?

(a)Lim (b)Mak (c)Nai
(d)Ong (e)Poh

Ans.(b)

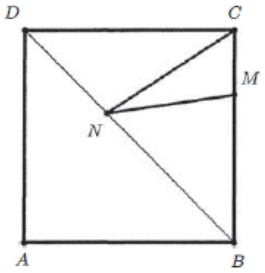
Q7. Given that ABC is a triangle with D being the midpoint of AC and E a point on CB such that CE=2EB. If AE and BD intersect at point F and the area of ΔAFB=1unit, find the area of ΔABC.



(a)3 (b) $\frac{10}{3}$ (c) $\frac{11}{3}$
(d)4 (e)5

Ans.(d)

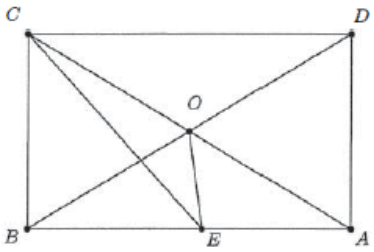
Q8. ABCD is a square with sides 8 cm. M is a point on CB such that CM=2cm. If N is a point on the diagonal DB, find the least value of CN+MN.



(a)8 (b)$6\sqrt{2}$ (c)10
(d)$8\sqrt{2}$ (e)12

Ans.(c)

Q9. ABCD is a rectangle whose diagonals intersect at point O. E is a point on AB such that CE bisects ∠BCD. If ∠ACE=15^0, find ∠BOE.



(a)60^0 (b)65^0 (c)70^0
(d)75^0 (e)80^0

Ans.(d)

Q10. Let *S* be the smallest positive multiple of 15, that comprises exactly 3*k* digits with *k* "0"s, *k* "3"s and *k* "8"s. find the remainder when *S* is divided by 11.

(a)0 (b)3 (c)5
(d)6 (e)8

Ans.(d)

Q11. Find the value of $\sqrt{9999^2 + 19999}$.

Ans. 10000

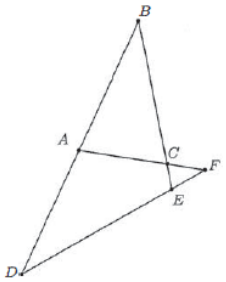
Q12. If the graphs of $y=x^2+2ax+6b$ and $y=x^2+2bx+6a$ intersect at only one point in the xy-plane, what is the x-coordinate of the point of intersection?

Ans.3

Q13.Find the number of multiples of 11 in the sequence 99, 100, 101, 102,….,20130.

Ans. 1822

Q14.In the figure below, BAD, BCE, ACF and DEF are straight lines. It is given that BA=BC, AD=AF, EB=ED. If $\angle BED=x^0$, find the value of x.



Ans. 108

Q15. If a = 1.69, b = 1.73, and c = 0.48, find the value of

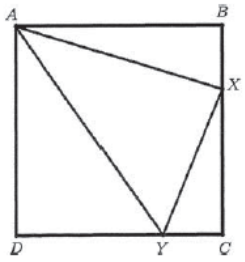
$$\frac{1}{a^2-ac-ab+bc}+\frac{2}{b^2-ab-bc+ac}+\frac{1}{c^2-ac-bc+ab}$$

Ans.20

Q16. Suppose that x_1 and x_2 are the two roots of the equation $(x-2)^2=3(x+5)$. What is the value of the expression $x_1x_2+x_1^2+x_2^2$?

Ans.60

Q17. Let ABCD be a square and X and Y be points such that the lengths of XY, AX and AY are 6,8, and 10 respectively. The area of ABCD can be expressed as $\frac{m}{n}$ units where m and n are positive integers without common factors. Find the value of $m+n$.



Ans.1041

Q18.Let x and y be real numbers satisfying the inequality $5x^2+y^2-4xy+24 \le 10x-1$. Find the value of x^2+y^2.

Ans. 125

Q19. A painting job can be completed by Team A alone in 2.5 hours or by Team B alone in 75 minutes. On one occasion, after Team A had completed a fraction $\frac{m}{n}$ of the job, Team B took over immediately. The whole painting job was completed in 1.5 hours. If m and n are positive integers with no common factors, find the value of $m+n$.

Ans.6

Q20. Let a, b, and c be real numbers such that $\frac{ab}{a+b}=\frac{1}{3}$, $\frac{bc}{b+c}=\frac{1}{4}$and $\frac{ca}{c+a}=\frac{1}{5}$. Find the value of $\frac{24abc}{ab+bc+ca}$.

Ans.4

Q21. Let x_1 and x_2 be two real numbers that satisfy $x_1x_2=2020$. What is the minimum value of $(x_1+x_2)^2$?

Ans.8080

Q22. Find the value of $\sqrt{45-\sqrt{2000}}+\sqrt{45+\sqrt{2000}}$

Ans.10

Q23. Find the smallest positive integer k such that $(k-10)^{4026} \ge 2013^{2013}$.

Ans.55

Q24. Let a and b be two real numbers. If the equation $ax+(b-3)=(5a-1)x+3b$ has more than one solution, what is the value of $100a+4b$?

Ans.19

Q25. Find the least positive integer n such that $2^8+2^{11}+2^n$ is a perfect square.

Ans.12

Q26. Find the units digit of $2013^1+2013^2+2013^3+\ldots+2013^{2013}$.

Ans.3