LET US DO SOME PROBLEMS- XXXX

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Imperial College London is one of the most prestigious colleges of the world. It has a great history behind it. Every student wants to study there. Some problems from the Question Paper of Entrance Examination are selected here for the students who are preparing for the Admission to this College so that they can understand the standard of the questions.

Q1. A regular dodecagon is a 12-sided polygon with all sides the same length and all internal angles equal. If I construct a regular dodecagon by connecting *12 equal-spaced* points on a circle of radius1, then the area of this polygon is

(a)$6+3\sqrt{3}$ (b) $2\sqrt{2}$
(c) $3\sqrt{2}$ (d) $3\sqrt{3}$ (e) 3

Ans.(c)

Q2. The positive number *a* satisfies $\int_0^a \left(\sqrt{x}+x^2\right)dx = 5$ if
(a)$a = \left(\sqrt{21}-1\right)^{1/3}$
(b)$a = \sqrt{3}$
(c)$a = 3^{2/3}$
(d) $a = \left(\sqrt{6}-1\right)^{2/3}$
(e) $a = 5^{2/3}$

Ans.(c)

Q3. Tangents to $y=e^x$ are drawn at (p, e^p) and (q,e^q). These tangents cross the *x-axis* at *a* and *b* respectively. It follows that, for all *p* and *q*,
(a)$ps=qb$ (b)$p-a<q-b$
(c)$p-a=q-b$ (d)$p-a>q-b$ (e)$p+q=a+b$

Ans.(c)

Q4. The area of the region bounded by the curve $y=e^x$, the curve $y=1-e^x$, and *y-axis* equals
(a)0 (b)1-ln2
(c)$\frac{1}{2}-\frac{1}{2}ln2$ (d)ln2-1 (e) $1-ln\frac{1}{2}$

[ln x is alternative notation for $\log_e x$]

Ans.(b)

Q5. The sum
$sin^2(1^0)+sin^2(2^0)+sin^2(3^0)+\cdots+sin^2(89^0)+sin^2(90^0)$
is equal to
(a)44 (b)$44\frac{1}{2}$
(c)45 (d) $45\frac{1}{2}$ (e)46

Ans.(d)

Q6. A sequence is defined by $a_0=2$ and then for $n \geq 1$, a_n is one more than the product of all previous terms (so $a_1=3$ and $a_2=7$ for example). It follows that for all $n \geq 1$,
(a)$a_n=4a_{n-1}-5$
(b)$a_n=a_{n-1}(a_{n-1}-1)+1$
(c)$a_n=2a_{n-1}(a_{n-1}-3)+7$
(d)$a_n = \frac{3}{2}n^2-\frac{1}{2}n+2$
(e)None of the above

Ans.(b)

Q7. Four distinct real numbers *a,b,c,* and *d* are used to define four points *A=(a,b), B=(b,c), C=(c,d), D=(d,a).* The quadrilateral *ABCD* has all four sides the same length

(a) If and only if $(a-b)^2=(c-d)^2$

(b) If and only if $(a-c)^2=(b-d)^2$

(c) If and only if $(a-d)^2=(b-c)^2$

(d) If and only if $a-b+c-d=0$

(e) For no values of *a,b,c,d*

Ans.(d)

Q8. A square has centre (3,4) and one corner at (1,5). Another corner is at

(a)(1,3) (b)(5,5)

(c)(4,2) (d)(2,2) (e)(5,2)

Ans.(d)

Q9. What is the value of

$\int_0^1 (e^x - x)(e^x + x)dx$?

(a)$\frac{3e^2-2}{6}$ (b) $\frac{3e^2+2}{6}$

(c) $\frac{2e^2-3}{6}$ (d) $\frac{3e^2-5}{6}$ (e) $\frac{3e^2+3}{6}$

Ans.(d)

Q10. The sum

$1 - 4 + 9 - 16 + \ldots + 99^2 - 100^2$

equals

(a)-101 (b)-1000

(c)-1111 (d)-4545 (e)-5050

Ans.(c)

Q11. The largest value achieved by $3\cos^2 x+2\sin x+1$ equals

(a) $\frac{11}{5}$ (b) $\frac{13}{3}$

(c) $\frac{12}{5}$ (d) $\frac{14}{9}$ (e) $\frac{12}{7}$

Ans.(b)

Q12. A line is tangent to the parabola $y=x^2$ at the point (a,a^2) where $a>0$. The area of the region bounded by the parabola, the tangent and the *x-axis* equals

(a) $\frac{a^2}{3}$ (b) $\frac{2a^2}{3}$

(c) $\frac{a^3}{12}$ (d) $\frac{5a^3}{6}$ (e) $\frac{a^4}{10}$

Ans.(c)

Q13. Which of the following expressions is equal to $\log_{10}(10\times9\times8\times\ldots\times2\times1)$?

(a)$1+5\log_{10}2+4\log_{10}6$

(b)$1+4\log_{10}2+2\log_{10}6+\log_{10}7$

(c)$2+2\log_{10}2+4\log_{10}6+\log_{10}7$

(d)$2+6\log_{10}2+4\log_{10}6+\log_{10}7$

(e)$2+6\log_{10}2+4\log_{10}6$

Ans.(c)

Q14. A cubic has equation $y=x^3+ax^2+bx+c$ and has turning points at (*1*, *2*)and (*3*,*d*) for some *d*. what is the value of *d*?

(a)-4 (b)-2

(c)0 (d)2 (e)4

And.(b)

Q15. In the range $-90^0 < x < 90^0$, how many values of x are there for which the sum to infinity

$$\frac{1}{tanx} + \frac{1}{tan^2x} + \frac{1}{tan^3x} + \cdots.$$

equals $tan\ x$?

(a)0 (b)1
(c)2 (d)3 (e)4

Ans.(b)

Q16. The inequalities $x^2+3x+2>0$ and $x^2+x < 2$ are met by all x in the region

(a) $x<-2$ (b)$-1<x<1$
(c)$x>-1$ (d)$x>-2$
(e) None of these

Ans.(b)

Q17. The smallest value of the function $f(x)=2x^3-9x^2+12x+3$ in the range $0 \le x \le 2$ is

(a)1 (b)3
(c)5 (d)7 (e)9

Ans.(b)

Q18. What is the reflection of the point (3,4) in the line $3x+4y=50$?

(a)(9,12) (b)(6,8)
(c)(12,16) (d)(16,12) (e)(4,3)

Ans.(a)

Q19. The equation $x^3-30x^2+108x-104=0$ has

(a) no real roots
(b)Exactly one real root
(c)Three distinct real roots
(d)A repeated root
(e) None of these is correct

Ans.(d)

Q20. The fact that $6 \times 7 = 42$ is a counter-example to which of the following statements?

(a)The product of any two odd integers is odd
(b)If the product of two integers is not a multiple of 4 then the integers are not consecutive
(c)If the product of two integers is a multiple of 4 then the integers are not consecutive
(d)Any even integer can be written as the product of two even integers
(e)None of the above

Ans.(b)