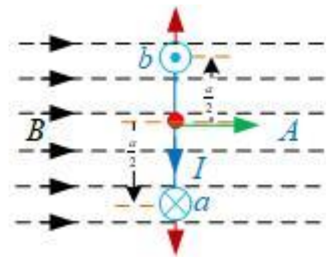
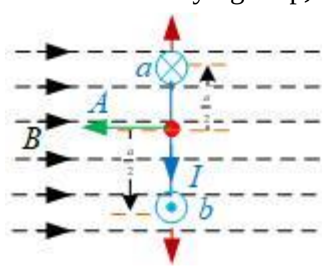
Electromagnetism: Magnetic Effect of Electric Current (Part III-Set 3)

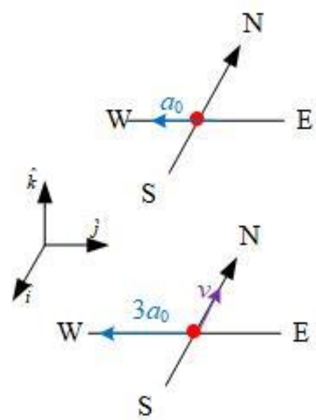
Selected Questions with Illustrations

Q-01	The torque on a current loop is zero if the angle between the positive normal and the magnetic field is either $\alpha = 0$ or $\alpha = 180^0$. In which of the two orientations the equilibrium is stable?
A-01	$\alpha = 0$
I-01	Despite net charge on a current carrying wire being zero, cloud of free electron in wire experience a unidirectional drift. This drift is responsible for current in the wire. When current carrying wire is placed in magnetic field, as per Bio-Savart's Law it produces a magnetic field around it. Interaction of these two magnetic field produces reorientation of magnetic field causing a force F on conductor as per Lorentz's Force Law expressed as $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = I\vec{l} \times \vec{B} \Rightarrow \vec{F} = IlB \sin\alpha\, \hat{n}$. Here, I is the current through wire, l is length of wire, B is magnetic field in which wire is placed, and α is the angle of magnetic field vector $\vec{B}$ w.r.t. length vector $\vec{l}$. The three parameters I, l, B are non- zero as per statement of problem. Therefore, if $\alpha \neq 0$, i.e. wire is not parallel to magnetic field then it will experience force. Thus, in the given system for force to be zero there are two possibilities –  **(i)** Angle $\alpha = 0 \Rightarrow \sin\alpha = 0$ between area vector $\vec{A}$ and magnetic field vector $\vec{B}$. This case is shown in figure where 'a' of the loop through which current is entering is below and 'b' of loop through which current is leaving is above. This is the natural position of least potential energy $U_i = 0$ of the current carrying loop**, a state of stable equilibrium.**  **(ii)**The angle $\alpha = \pi \Rightarrow \sin\alpha = 0$. his case is shown in figure where 'a' of the loop through which current is entering is above and 'b' of loop through which current is leaving is below. Position of current carrying loop in this case requires work to be done by external force to rotate coil for position in case (i). Thus potential energy of the current carrying loop in this case $U_i > 0$. Hence, current carrying loop in this case despite forces on the loop being in equilibrium, **the loop is in instable equilibrium.** Thus, **answer is $\boldsymbol{\alpha = 0}$** **N.B.:** Area vector $\vec{A}$ is of magnitude equal to the area A under consideration and in a direction such that direction of perimeter is anti-clockwise direction. This is consistent with angles measured (+)ve in anticlockwise direction
Q-02	Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame S and $\vec{E}'$ and $\vec{B}'$ in another frame S' moving w.r.t. S with a velocity $\vec{v}$. Two of the following equations are wrong. Identify them (a) $B'_y = B_y + \frac{vE_z}{c^2}$ (b) $B'_y = E_y - \frac{vB_z}{c^2}$ (c) $B'_y = B_y + vE_z$ (d) $B'_y = E_y + vB_z$
A-02	(b), (c)

I-02	As per Electromagnetic Field Theory, magnetic field $\vec{B} = B\hat{b}$ and Electric field $\vec{E} = E\vec{e}$ produces electromagnetic wave $c\hat{v}$ which propagates with velocity c in direction $\hat{v} \perp \vec{e}$ && $\hat{v} \perp \hat{b}$, while $\vec{e} \perp \hat{b}$. Dimensionally, $[E] = \mathrm{MLT^{-3}I^{-1}}$ …(1), $[B] = \mathrm{MI^{-1}T^{-2}}$…(2) and $[v] = \mathrm{LT^{-1}}$…(3). Each of the option is being analyzed dimensionally – **Option (a):** $B'_y = B_y + \frac{vE_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \mathrm{MI^{-1}T^{-2}}$. Since both the addends on the RHS have same dimensions hence $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}}$. *Dimensionally,* $[LHS] = [RHS]$, *these are not wrong. Hence, as desired option (a) is incorrect.* **Option (b):** $B'_y = E_y - \frac{vB_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{MI^{-1}T^{-2}}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.* **Option (c):** $B'_y = B_y + vE_z \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + (\mathrm{LT^{-1}}) \times (\mathrm{MLT^{-3}I^{-1}})$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{ML^2I^{-1}T^{-4}}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.* **Option (d):** $B'_y = E_y + vB_z \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + (\mathrm{LT^{-1}}) \times (\mathrm{MI^{-1}T^{-2}})$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{MLI^{-1}T^{-3}}$. Both the addends on the RHS have same dimensions hence $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}}$. *Dimensionally,* $[LHS] = [RHS]$, *these are not wrong. Hence, as desired option (a) is incorrect.* Hence, **answer is option (b) and (c)** **N.B.:** This problem requires understanding of electromagnetic waves. Yet, despite E, B and v being discretely different physical quantities, correctness of relations between them has been solved dimensionally.
Q-03	Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame S and $\vec{E}'$ and $\vec{B}'$ in another frame S' moving w.r.t. S with a velocity $\vec{v}$. Two of the following equations are wrong. Identify them (a) $B'_y = B_y + \frac{vE_z}{c^2}$ (b) $B'_y = E_y - \frac{vB_z}{c^2}$ (c) $B'_y = B_y + vE_z$ (d) $B'_y = E_y + vB_z$
A-03	(b), (c)
I-03	As per Electromagnetic Field Theory, magnetic field $\vec{B} = B\hat{b}$ and Electric field $\vec{E} = E\vec{e}$ produces electromagnetic wave $c\hat{v}$ which propagates with velocity c in direction $\hat{v} \perp \vec{e}$ && $\hat{v} \perp \hat{b}$, while $\vec{e} \perp \hat{b}$. Dimensionally, $[E] = \mathrm{MLT^{-3}I^{-1}}$ …(1), $[B] = \mathrm{MI^{-1}T^{-2}}$…(2) and $[v] = \mathrm{LT^{-1}}$…(3). Each of the option is being analyzed dimensionally – **Option (a):** $B'_y = B_y + \frac{vE_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}} + \mathrm{MI^{-1}T^{-2}}$. Since both the addends on the RHS have same dimensions hence $[\mathrm{RHS}] = \mathrm{MI^{-1}T^{-2}}$. *Dimensionally,* $[LHS] = [RHS]$, *these are not wrong. Hence, as desired option (a) is incorrect.* **Option (b):** $B'_y = E_y - \frac{vB_z}{c^2} \Rightarrow [\mathrm{LHS}] = \mathrm{MI^{-1}T^{-2}}$ && $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \frac{(\mathrm{LT^{-1}})\times(\mathrm{MLT^{-3}I^{-1}})}{(\mathrm{LT^{-1}})^2}$. The RHS leads to $[\mathrm{RHS}] = \mathrm{MLT^{-3}I^{-1}} + \mathrm{MI^{-1}T^{-2}}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.*

	Option (c): $B'_y = B_y + vE_z \Rightarrow [\text{LHS}] = \text{MI}^{-1}\text{T}^{-2}$ && $[\text{RHS}] = \text{MI}^{-1}\text{T}^{-2} + (\text{LT}^{-1}) \times (\text{MLT}^{-3}\text{I}^{-1})$. The RHS leads to $[\text{RHS}] = \text{MLT}^{-3}\text{I}^{-1} + \text{ML}^2\text{I}^{-1}\text{T}^{-4}$. Both the addends on RHS have unequal dimensions and hence they can be added. *This make statement at option (b) wrong. Hence as desired option (b) is correct.* **Option (d):** $B'_y = E_y + vB_z \Rightarrow [\text{LHS}] = \text{MI}^{-1}\text{T}^{-2}$ && $[\text{RHS}] = \text{MLT}^{-3}\text{I}^{-1} + (\text{LT}^{-1}) \times (\text{MI}^{-1}\text{T}^{-2})$. The RHS leads to $[\text{RHS}] = \text{MLT}^{-3}\text{I}^{-1} + \text{MLI}^{-1}\text{T}^{-3}$. Both the addends on the RHS have same dimensions hence $[\text{RHS}] = \text{MI}^{-1}\text{T}^{-2}$. *Dimensionally,* $[LHS] = [RHS]$, *these are not wrong. Hence, as desired option (a) is incorrect.* Hence, **answer is option (b) and (c)** **N.B.:** This problem requires understanding of electromagnetic waves. Yet, despite E, B and v being discretely different physical quantities, correctness of relations between them has been solved dimensionally.
Q-04	A 10 g bullet having a charge 4.00mC is fired at a speed 270 m/s in a horizontal direction. A vertical magnetic field of 500 μT exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m. Make appropriate approximations.
A-04	3.7×10^{-6}m
I-04	 Given that a bullet of mass $m = 0.01$ kg carries a charge $q = 4.00 \times 10^{-6}$ C is fired along horizontal direction with a speed $v = 270$ m/s. There is a vertical magnetic field $B = 500 \times 10^{-6}$ T. For convenience initial position of the particle is taken at origin O and its motion along $(\hat{\imath} - \hat{\jmath})$ velocity vector as $\vec{v} = v\hat{\jmath}$…(1) as shown in the figure. Accordingly, $\vec{B} = B\hat{k}$…(2). As per Lorentz's Force Law $\vec{F} = q\vec{v} \times \vec{B}$…(3). Combining (1), (2) and (3), $\vec{F} = q(v\hat{\jmath} \times B\hat{k}) \Rightarrow \vec{F} = qvB(\hat{\jmath} \times \hat{k}) \Rightarrow \vec{F} = qvB\hat{\imath}$…(4), is arrived at using principle of cross-product of vectors. Therefore, acceleration of the particle taking (4), as per mechanics, is $\vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{a} = \frac{qvB}{m}\hat{\imath}$…(5). Using available data, $\vec{a} = \frac{(4.00\times10^{-6})(270)(500\times10^{-6})}{0.01}\hat{\imath} \Rightarrow \vec{a} = 5.4 \times 10^{-5}\hat{\imath}$ m/s²…(6). Rest of the problem is simple application of concepts of projectile motion in this case. While particle is accelerated with magnetic force along $\hat{\imath}$, its travel of $y = 100$ m along $\hat{\jmath}$ is un-accelerated. Hence, hence time t taken in this travel, using available data, is $t = \frac{y}{v} \Rightarrow t = \frac{100}{270} \Rightarrow t = \frac{10}{27}$ s…(7). Motion of the particle with acceleration $\vec{a}$ with initial velocity $u = 0$ in time t is deflection $x = BC = OA$ of the particle along $\hat{\imath}$. This can be determined with equation of motion, $x = 0 \times t + \frac{at^2}{2}$…(8). Using available data in (8) we have $x = \frac{(5.4\times10^{-5})\left(\frac{10}{27}\right)^2}{2} \Rightarrow x = \mathbf{3.70 \times 10^{-6}}$**m is the answer.** **N.B.:** This problem involves integration of concepts of mechanics with Lorentz' Force Law.
Q-05	When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards the west. When it is projected towards north with a speed v_0 , it moves with an initial acceleration $3a_0$ towards west. Find the electric field and maximum possible magnetic field in the room.
A-05	$\frac{ma_0}{e}$ towards west, $\frac{2ma_0}{ev_0}$ downward

I-05



Given that in a room there are electric and magnetic fields. Acceleration of proton in two different cases proton is given as shown in the figure. Reference unit direction vectors are also indicated in the figure to facilitate analysis.

Motion of a proton of mass m and charge e can be analyzed in accordance with Lorentz's Force Law $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \Rightarrow \vec{a} = \frac{e}{m}\vec{E} + \frac{evB}{m}\sin\theta\ \hat{n}$…(1). Here, θ is the angle of magnetic field vector wire length $\vec{B}$ w.r.t. velocity vector $\vec{v}$, while unit vector $\hat{n}$ is perpendicular to the plane containing vectors $\vec{v}$ & $\vec{B}$.

Given are two cases as under -

Case 1: When proton is released from the state of rest $v = 0$, acceleration of the particle as per (1) would be $\vec{a}_1 = \frac{e}{m}\vec{E}$…(2). It is given that acceleration is towards west, $\vec{a}_1 = a_0(-\hat{j})$…(3). Combining (2) and (3) we get $\frac{e}{m}\vec{E} = a_0(-\hat{j}) \Rightarrow \vec{E} = \frac{a_0 m}{e}(-\hat{j})$…(4). Using reference vectors **electric field is of magnitude $\boldsymbol{\frac{a_0 m}{e}}$ towards west.**

Case 2: When proton projected towards north with a velocity $\vec{v} = v_0(-\hat{\imath})$, it experiences an acceleration $\vec{a}_2 = 3a_0(-\hat{j})$…(5). As per (1) together with (4) is $\vec{a}_2 = \frac{e}{m}\left(\frac{a_0 m}{e}\right)(-\hat{j}) + \frac{evB}{m}\sin\theta\ \hat{n}$…(6). Combining (5) and (6), $(-)3a_0\hat{j} = (-)a_0\hat{j} + \frac{qeB}{m}\sin\theta\ \hat{n} \Rightarrow \frac{ev_0 B}{m}\sin\theta\ \hat{n} = (-)2a_0\hat{j}$. It leads to $B\,\hat{n} = \frac{2ma_0}{ev\sin\theta}(-\hat{j})$…(7).

Going back to discussions following (1) and that velocity v is along $(-\hat{\imath})$the magnetic field $\vec{B}$ would be on $(\hat{\imath} - \hat{k})$ plane. Accordingly, angle θ is of magnetic field with velocity in $(\hat{\imath} - \hat{k})$. The equation (8), where parameters m, e, a_0 and $, v_0$ are constant, be written as $B = K\frac{1}{\sin\theta}$. Therefore, for maximum B, let us apply concept of maxima-minima $\frac{dB}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left(\frac{1}{\sin\theta}\right) = 0$…(8) Now substitute $u = \sin\theta \Rightarrow \frac{du}{d\theta} = \frac{d}{d\theta}\sin\theta \Rightarrow \frac{du}{d\theta} = \cos\theta$. Manipulating (8) $\frac{d}{d\theta}\left(\frac{1}{u}\right) = \frac{d}{du}\left(\frac{1}{u}\right) \times \frac{du}{d\theta}$. It further leads to $\frac{d}{d\theta}\left(\frac{1}{u}\right) = (-)\frac{1}{u^2} \times \cos\theta \Rightarrow \frac{dB}{d\theta} = (-)\frac{\cos\theta}{\sin^2\theta}$…(9). Further, $\frac{\cot\theta}{\sin\theta} = 0 \Rightarrow \cot\theta = 0$. It is a trigonometric equation and its principal solution for either maxima or minima is $\theta \pm \frac{\pi}{2}$. It requires to choose among the two possible values of θ for maximum value of B. This is ascertained by taking second derivative of (8). If $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) > 0$ the its solution among the two values will give minimum B, else if $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) < 0$ then minima.

Accordingly, $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{d}{d\theta}\left(-\frac{\cot\theta}{\sin\theta}\right) \Rightarrow \frac{d}{d\theta}\left(-\frac{\cot\theta}{\sin\theta}\right) = (-)\frac{\sin\theta\left(\frac{d}{d\theta}\cot\theta\right) - \cot\theta\left(\frac{d}{d\theta}\sin\theta\right)}{\sin^2\theta}$. It, further, solves into $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = (-)\frac{\sin\theta \times (-\mathrm{cosec}^2\theta) - \cot\theta \times \sin\theta}{\sin^2\theta} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{\mathrm{cosec}\,\theta + \cot\theta \times \sin\theta}{\sin^2\theta}$…(10).

This is where value of $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right)$ in (10) needs to be examined for solution $\theta \pm \frac{\pi}{2}$of obtained from (9), Taking each of the values-

(i) $\theta = (+)\frac{\pi}{2}$: Then, $\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{1 - 0 \times 1}{1} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = 1 \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) > 0$ is condition of minima.

(ii) $\boldsymbol{\theta = (-)\frac{\pi}{2}}$: Then, $\boldsymbol{\frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = \frac{-1 - 0 \times (-1)}{(-1)^2} \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) = -1 \Rightarrow \frac{d}{d\theta}\left(\frac{dB}{d\theta}\right) < 0}$ is condition of maxima, as desired

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As discussed following (1) above, θ is angular displacement of vector $\vec{B}$ w.r.t. $\vec{v}$ and that the angle is (+)ve in anticlockwise direction, while it is (-)ve in clockwise direction. Thus, going back to the figure magnetic field $\vec{B}$ will be along $\left(\hat{k}\right)$ i.e. downward .

Thus, answers are $E = \frac{ma_0}{e}$ toward west and $B = \frac{2ma_0}{ev_0}$ downward.

N.B.: This problem integrates concepts of electromagnetic force along with mathematics of vector algebra, trigonometric equations, maxima-minima, direction of angular displacement, It is a good example to appreciate beauty of mathematics in analysis of physical situations correctly, and in an unambiguous manner. Proficiency and confidence at it is acquired through understanding of concepts and its practice in problem solving.
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<tr><td>Q-06</td><td>
A hypothetical magnetic field existing in a region is given by $\vec{B} = B\vec{e}_r$, where $\vec{e}_r$ denotes unit vector along the radial direction. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the X-Y plane and the center at $(0,0,d)$. Find the magnitude of the magnetic force acting on the loop.
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<tr><td>A-06</td><td>
$\frac{2\pi\alpha^2 iB}{\sqrt{a^2+d^2}}$ downward
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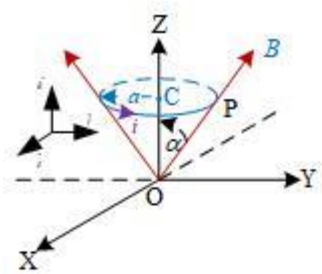
<tr><td>I-06</td><td>
Given is a circular loop placed parallel to X-Y plane with it center $\mathrm{C}(0,0.d)$, it implies $\mathrm{OC} = d\ldots(1)$ and a hypothetical magnetic field at every point on the perimeter of the loop $\vec{B} = B_0\vec{e}_r\ldots(2)$, here unit vector is along the line joining origin O and point of consideration on the perimeter of the loop; in case of point P it is $\vec{e}_r||\mathrm{OP}$. Correspondence of X,Y,Z axes with unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ is shown in the figure.



It is seen that angle α is between vectors $\hat{k}$ & $\hat{e}_r$ and is uniform at every point on the perimeter of the circular loop.

Since loop is not a straight wire hence force on the loop can be determined by initially taking force experienced by an element length $\Delta\vec{l} = \Delta\vec{\theta} \times \vec{r} \Rightarrow \Delta\vec{l} = \Delta\vec{\theta} \times r\hat{r} \Rightarrow \Delta\vec{l} = r\left(\Delta\vec{\theta} \times \hat{r}\right) \Rightarrow \Delta\vec{l} = r\left(\Delta\theta\hat{k} \times \hat{r}\right)\ldots(3)$. here $r = \mathrm{OP}$ and $\vec{\theta}$ is along $\hat{k}$.

Accordingly, $\Delta\vec{l} = r\Delta\theta\left(\hat{k} \times \hat{r}\right) \Rightarrow \Delta\vec{l} = \Delta l\left(\hat{k} \times \hat{r}\right)\ldots(4)$.

In the given geometry, $\vec{B} = B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k}\ldots(5)$.

Magnetic force as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \Delta\vec{F} = \left(i\Delta\vec{l}\right) \times \vec{B} \Rightarrow \Delta\vec{F} = i\left(\Delta\vec{l} \times \vec{B}\right)\ldots(6)$. Here, geometrically $\vec{B} = B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k}\ldots(7)$

Combining (4), (5) and (6) we have $\Delta\vec{F} = i\left(\Delta l\left(\hat{k} \times \hat{r}\right) \times \left(B\sin\alpha\,\hat{r} + B\cos\alpha\,\hat{k}\right)\right)\ldots(8)$

Eqn. (8) leads to $\Delta\vec{F} = \Delta liB\left(\sin\alpha\left(\hat{k} \times \hat{r} \times \hat{r}\right) + \cos\alpha\left(\hat{k} \times \hat{r} \times \hat{k}\right)\right)$,,,(9). This turns out to be problem of triple cross product of vectors. Instead considering symmetry of the geometry, as shown in figure, let us simplify using (3) at P $\Delta\vec{l} = a\Delta\theta(-\hat{\imath})$ and $\hat{r} = \hat{\jmath}$. With this, combining (6) and (7), we have –
$$\Delta\vec{F} = i\left(a\Delta\theta(-\hat{\imath})\right) \times \left(B\sin\alpha\,\hat{\jmath} + B\cos\alpha\,\hat{k}\right) \Rightarrow \Delta\vec{F} = (-)iaB\Delta\theta\left(\sin\alpha\left(\hat{\imath} \times \hat{\jmath}\right) + \cos\alpha\left(\hat{\imath} \times \hat{k}\right)\right)$$
$$\Delta\vec{F} = (-)iaB\Delta\theta\left(\sin\alpha\,\hat{k} + \cos\alpha\,\hat{\jmath}\right) \Rightarrow \Delta\vec{F} = \left(iaB\sin\alpha\left(-\hat{k}\right) + iaB\cos\alpha\,\hat{\jmath}\right)\Delta\theta$$
$$\Delta\vec{F} = \Delta F_a\left(-\hat{k}\right) + \Delta F_r(-\hat{r}),\ \text{here}\ \Delta F_a = iaB\sin\alpha\,\Delta\theta\ \text{and}\ \Delta F_r = iaB\cos\alpha\,\Delta\theta\ldots(9)$$
An observation of the symmetrical geometry t is to be noted ΔF_r component acting along $\hat{\jmath} \to \hat{r}$ would mutually cancel with geometrically opposite points leading to $F_r = \oint \Delta F_r = 0\ldots(10)$.
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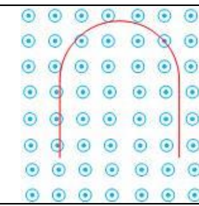
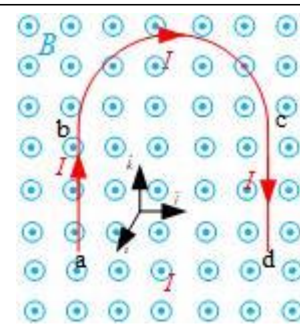
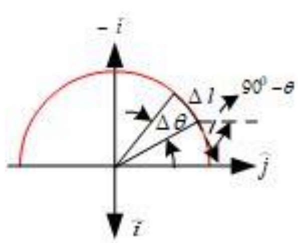
<tr><td></td><td>In respect of axial force ΔF_a is along $(-\hat{k})$ i.e. downward and net force over the circular loop would be $F_a = \oint \Delta F_a \Rightarrow F_a = iaB\sin\alpha \oint \Delta\theta \Rightarrow F_a = 2\pi iaB\sin\alpha$…(11). Going back to the geometry $\sin\alpha = \frac{a}{\sqrt{a^2+d^2}}$ …(12). Combining (11) and (12), together with the direction discussed above, net force on the circular loop is $F = F_a = 2\pi iaB \times \frac{a}{\sqrt{a^2+d^2}} \Rightarrow \boldsymbol{F} = \frac{\boldsymbol{2\pi ia^2B}}{\sqrt{\boldsymbol{a^2+d^2}}}$ downward is the answer.

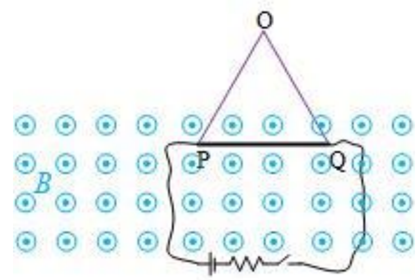
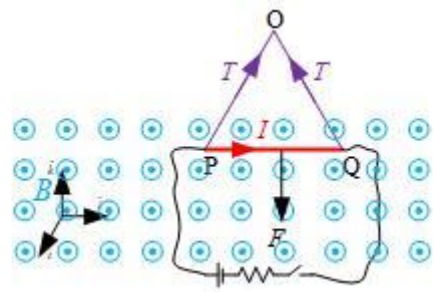
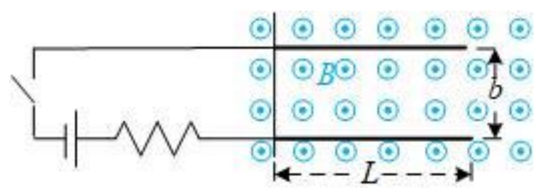
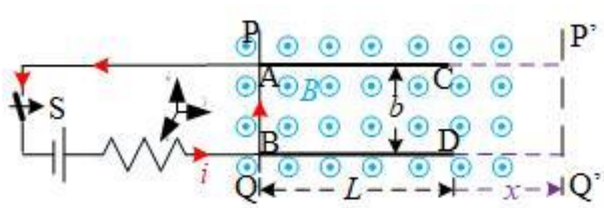
N.B.: Though this problem is for a hypothetical magnetic field, yet it is a gives good practice to gain proficiency in handling three dimensional vectors. Further, mathematics is an effective analytical tool problem can and should be simplified using symmetries wherever possible.</td></tr>
<tr><td>Q-07</td><td>A rigid wire consists of a semicircular portion of radius R and two straight sections as shown in the figure. The wire is partially immersed in a perpendicular magnetic field B as shown in the figure. Find the magnetic force on the wire if it carries a current I.
</td></tr>
<tr><td>A-07</td><td>$2IRB$, upward in the figure</td></tr>
<tr><td>I-07</td><td>Given system is shown in the figure. A rigid wires is shaped such that it has two straight and parallel portions of equal lengths cb and cd $\vec{l}_{ab} = l(-\vec{\imath})$…(1), and $\vec{l}_{cd} = l\vec{\imath}$…(2) respectively. The length vectors, though parallel are taken in direction of currents and unit-direction vectors, as shown in the figure. These two portions are connected through a portion bc in semicircular shape of radius R. Vectorially, a small length of the arc $\Delta\vec{l}_{bc} = R\hat{r} \times \Delta\theta(-\hat{k})$…(3); here $\hat{r}$ unit-direction vector of the element $\Delta\vec{l}_{bc}$and unit-direction vector of $\Delta\theta$ is taken along $(-\hat{k})$ since current in the semicircular portion is in clockwise direction.

The wire is taken to be on $(\hat{\imath} - \hat{\jmath})$ plane while magnetic field, as shown in the figure, is $\vec{B} = B\hat{k}$…(4).

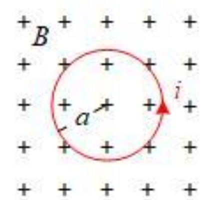
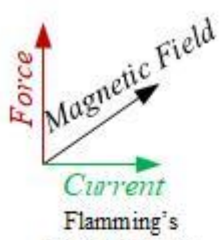
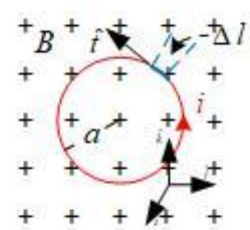
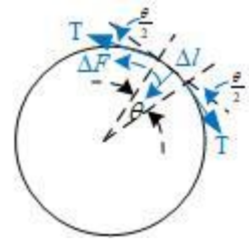
Magnetic force experience by a conductor as per Lorentz's Force Law is $\Delta\vec{F} = (q\vec{v}) \times \vec{B} \Rightarrow \vec{F} = (i\vec{\Delta l}) \times \vec{B} \Rightarrow \Delta\vec{F} = i(\Delta\vec{l} \times B\hat{k}) \Rightarrow \Delta\vec{F} = iB(\vec{l} \times \hat{k})$…(5).

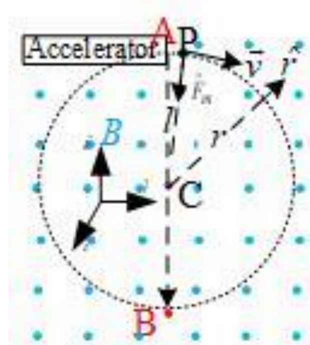
Combining above equations it leads to –

$$\vec{F} = \vec{F}_{ab} + \vec{F}_{bc} + \vec{F}_{ca}$$
$$\vec{F} = IB\left(l(-\vec{\imath}) \times B\hat{k} + \int_\pi^0 \left(R\hat{r} \times d\theta(-\hat{k})\right) \times \hat{k} + l(\vec{\imath}) \times \hat{k}\right)$$
$\vec{F} = (-)IRB\left(\int_\pi^0 (\hat{l}d\theta) \times \hat{k}\right) \Rightarrow \vec{F} = (-)IRB\left(\int_\pi^0 (\hat{l}d\theta) \times \hat{k}\right)$; here unit vector $\vec{l} = \cos(90^0 - \theta)\hat{\jmath} - \sin(90^0 - \theta)\hat{\imath}$.

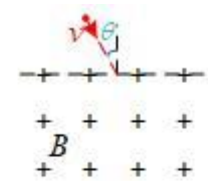
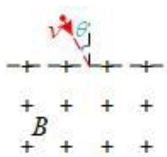
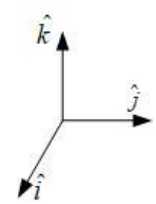
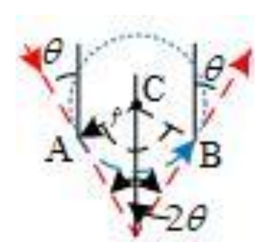
Thus, $\vec{F} = (-)IRB\left(\int_\pi^0 ((\sin\theta\,\hat{\jmath} - \cos\theta\,\hat{\imath})d\theta) \times \hat{k}\right)$. It leads to -
$$\vec{F} = (-)IRB\left(\int_\pi^0 \left(\left((\hat{\jmath} \times \hat{k})\sin\theta - (\hat{\imath} \times \hat{k})\cos\theta\right)d\theta\right)\right) \Rightarrow \vec{F} = (-)IRB\left(\int_\pi^0 ((\hat{\imath}\sin\theta + \hat{\jmath}\cos\theta\,\hat{\imath})d\theta)\right)$$
$\vec{F} = (-)IRB([\hat{\imath}\sin\theta - \hat{\jmath}\cos\theta]_\pi^0)\hat{\imath} \Rightarrow \vec{F} = IRR[\cos 0 - \cos\pi]\hat{\imath} \Rightarrow \vec{F} = 2IRB\hat{\imath}$, Thus, force is $2IRB$ downward is the answer.

N.B.: It is an example of proficiency in analysis using mathematics as an unambiguous tool of clarity.</td></tr>
</table>

Q-08	Figure shows that a rod PQ of length 20.0 cm and mass 200 g suspended through a fixed point O by two threads of length of length 20.0 cm each. A magnetic field of strength 0.500 T exists in the vicinity of the wire PQ as shown in the figure. The wires connecting PQ with the battery are loose and exert no force on PW.  (a) Find the tension in the threads when switch S is open. (b) A current of 2.0 A is established when the switch S is closed. Find the tension in the threads now.
A-08	(a) 1.2 N (b) 1.3 N
I-08	Given system is shown in the figure and with given data ΔOPQ is equilateral of side $l = 0.20$ m. For convenience 3D unit vectors are shown in the figure. Hence, $2T\sin\frac{\pi}{3} = F \Rightarrow 2T\left(\frac{\sqrt{3}}{2}\right) = F \Rightarrow T = \frac{F}{\sqrt{3}}$.  Given that mass of wire $m = 0.200$ kg and $\vec{B} = 0.500\hat{\imath}$ T, and acceleration due to gravity is not specified it's magnitude is taken as $g = 10$ m/s^2. It leads to $\vec{g} = 10(-\hat{k})$ ms^2. Both the parts are solved below – **Part (a):** When switch is open $\vec{F} = \vec{F}_g = m\vec{g}$; using given data $\vec{F} = 0.200 \times 10(\hat{\imath}) \Rightarrow F = 2.00$ N$\Rightarrow T = \frac{2.00}{\sqrt{3}} \Rightarrow T = 1.15$ N or $T = 1.2$ N **Part (b):** When switch is closed net force on wire would be $\vec{F} = \vec{F}_g + \vec{F}_m$. Here, $\vec{F}_m = (i\vec{l}) \times \vec{B} \Rightarrow \vec{F}_m = i(l\hat{\jmath} \times B\hat{k}) \Rightarrow \vec{F}_m = ilB\hat{\imath}$. Thus, using available data $\vec{F} = 2.00\hat{\imath} + 2.0 \times 0.20 \times 0.500\hat{\imath} \Rightarrow \vec{F} = 2.2\hat{\imath}$ N. Hence, tension in the strings would be $T = \frac{2.2}{\sqrt{3}} \Rightarrow T = 1.3$ N. Hence, **answers are (a) $\mathbf{1.2}$ N and (b) $\mathbf{1.3}$ N.** **N.B.:** Reporting of answers is using principle of significant digits.
Q-09	Two metal strips, each of length L, are clamped parallel to each other on a horizontal floor with a separation b between them. A wire of mass m lies on them perpendicularly as shown in the figure. A vertically upward magnetic field of strength B exists in the space. The metal strips are smooth but coefficient of friction between the wire and the floor is μ. A current i is established when switch S is closed at the instant $t = 0$.  Discuss the motion of wire after switch is closed. How far away from the strp will the wire reach?
A-09	$\frac{ilbB}{\mu mg}$
I-09	For convenience of analysis unit vectors in 3D asre shown in the figure. Given system is placed in $(\hat{\imath} - \hat{\jmath})$ plane and magnetic field is $\vec{B} = B\hat{k}$...(1). At $t = 0$, switch S is closed wire PQ is at ends AB of the two strips AC‖BD clamped at a searation $\vec{l} = b(-\hat{\imath})$. It will establish a current i in the circuit as shown in the figure. This current flows in portion BA of the wires from B to A whose length is $l = b$...(2), i.e. separation between the two strips ACand BD, each of length L that are clamped. The strips AC and BD are firction less, but while after traversing the length 

	As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F} = i\vec{l} \times \vec{B}$. Using the avaliable data $\vec{F} = i\left(b(-\hat{\imath}) \times \left(B\hat{k}\right)\right) \Rightarrow \vec{F} = ibB\left((-)\hat{\imath} \times \hat{k}\right) \Rightarrow \vec{F} = ibB\hat{\jmath}$...(3). Thus wire of mass m would experience a force and acceleration $F = ibB \Rightarrow a = \frac{F}{m} \Rightarrow a = \frac{ibB}{m}$...(4) along right side i,e, toward ends C-D. Strips are of hiher crossection and hence considered to be of negligible resistance. Therefore, while wire under amgnetic force would slip along the length L of the strip there would be no change of current and consequently force F and acceleration a would remain constant. Thus velocity attained by the wire, starting from state of rest frpm position PQ with $u = 0$, as it reaches position P'Q' and touches the floor with a velocity v, as per 3rd equation of motion, $v^2 = u^2 + 2as$...(5). It, wiith available data, leads to $v^2 = 0 + 2\left(\frac{ibB}{m}\right)L \Rightarrow v^2 = \frac{2ibBL}{m}$...(6). As soon as wire touches ground having coefficient of friction μ, it experiences a frictional force $f = -\mu mg$ ...(7), here is acceleration due to gravity. Thus wire would experience a frictional$a_f = \frac{f}{m} \Rightarrow a_f = (-)\frac{\mu mg}{m}$. It leads to $a_f = (-)\mu g$...(8). Again applying (5), in this case with $u^2 = v^2 = \frac{2ibBL}{m}$, $v^2 = 0$ and $a_f = (-)\mu g$, distance $s = x$ travelled by the wire, as shown in the figure, until it stops is $0 = \frac{2ibBL}{m} + 2\left((-)\mu g\right)x \Rightarrow x = \frac{\frac{2ibBL}{m}}{2\mu g} \Rightarrow \boldsymbol{x = \frac{ibBL}{\mu mg}}$ **is the answer.** **N.B.:** This problems integrates electromagnetism with mechanics.
Q-10	Figure shows a cirular wire-loop of a radius a, carrying a current i, placed in a parpendicular magnetic field B. (a) Consider a small part Δl of the wire. Find the force on this paet of the wire exerted by the magnetic field. (b) Find the force of compression in the wire. 
A-10	(a) $i\Delta lB$ towards the center (b) iaB
I-10	As per Lorentz's Force Law current through wire length $\vec{l}$ will produce a amgnetic force $\vec{F} = i\vec{l} \times \vec{B}$. In this case a small length of the circular loop $\Delta\vec{l} = \Delta l\hat{t}$ and magnetic field $\vec{B} = B\left(-\hat{k}\right)$. Accordingly, $\Delta\vec{F} = i\left((\Delta l\hat{t}) \times B\left(-\hat{k}\right)\right)$...(1). Here, unit tangent vector for Δl is $\hat{t}$ and is $\perp$ to $\vec{B}$. Hence, asper Flemmings Left Hand Rule, as shown in the figure, is $\Delta\vec{F} = i\Delta lB(-\hat{r})$...(2). Since, $\hat{r}$ is unit vector along radius i.e. ouward and hence magnetic force along $(-\hat{r})$ toward the center of the cirle. Thus, **answer of part (a) wire is $\boldsymbol{i\Delta lB}$ towards the center.**  Flamming's Left Hand Rule  **Part (b)** requires to determine fore of compression on wire and is being analyzed with priniples of statics of forces. As determined in part (a) force on small part of the circualr wire which subtends an angle θ at its center O is $\Delta F = i\Delta lB$ towards the center as per (2). This force is result of tensile tension T experienced by the small part of the wire. Geometrically tension T is at an angle $\left(90^0 - \frac{\theta}{2}\right)$ with the force ΔF. Therefore, vectorially $\Delta F = 2T\cos\left(90^0 - \frac{\theta}{2}\right) \Rightarrow \Delta F = 2T\sin\frac{\theta}{2}$....(3) 

	Since, $\Delta l \ll\Rightarrow \theta \to 0 \Rightarrow \sin\theta \to \theta$. Therefore, $\sin\frac{\theta}{2} \to \frac{\theta}{2}$…(4). It leads to $\Delta F = 2T\frac{\theta}{2} \Rightarrow \Delta F = T\theta$…(5). Combining (2) and (5), $i\Delta lB = T\theta$. Since length of the small part is $\Delta l = a\theta$. It leads to $T\theta = i(a\theta)B$. It leads to $\boldsymbol{T = iaB}$ **is answer of the part (b).** **N.B.:** It is the application of principle of Hoop Stress, in mechanics, into electromagnetism.
Q-11	A proton describes a circle of radius 1 cm in a magnetic field of strength 0.10 T. What would be the radius of the circle by an α-particle moving with the same speed in the same magnetic field?
A-11	2 cm
I-11	Given that radius of circle described by a proton in magnetic field $B = 0.10$ T is $r_p = 1$ cm, It is required to find radius r_α of an α-particle moving with the same speed in the same magnetic field. This problem involves concept of magnetic force as per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow F = qvB\sin\theta\,\hat{n}$…(1) and mechanics of uniform cicular motion $F = \frac{mv^2}{r}$…(2). For uniform cicular motion combining (1) and (2) $qvB\sin\theta = \frac{mv^2}{r} \Rightarrow \frac{v}{B\sin\theta} = \frac{qr}{m}$…(3). With identical $\vec{v}$ and $\vec{B}$ for proton and α-particle L.H.S is same for both the paticles accordingly, $\frac{q_p r_p}{m_p} = \frac{q_\alpha r_\alpha}{m_\alpha} \Rightarrow r_\alpha = \left(\frac{q_p m_\alpha}{q_\alpha m_p}\right) r_p$…(4) Given that $r_p = 1$ cm , and we know that charge of proton $q_p = e$ and $q_\alpha = 2e$, while taking mass of proton $m_p = m$, mass of alpha particle is $m_\alpha = 4m$. Accordingly, $r_\alpha = 1 \times \frac{e\times 4m}{2e\times m} \Rightarrow r_\alpha =$ **2cm is the answer**. **N.B.:** In the problem value of B is notional and is not rrequired in arriving at results. Secondly, though radius of proton is given in CGS unit, deliberately it has not been converted in SI, because what is required to be detrmined is another radius. Thirdly, all quantities of coefficient in (4) are ratios of identical quantities of the two particles. Thus, this problem gets automatically simplified, without involving apparent calculations.
Q-12	Protons having kinetic energy K emerges from an accelerator as a narrow beam. The beam is bent by a perpendicular magnetic field so that it just misses a plane target kept at a distance l in front of the accelerator. Find the magnetic field.
A-12	$\frac{\sqrt{8mK}}{el}$ where m is mass of proton,
I-12	For convenience of analysis, unit vectors in 3D are shown in the figure. Protons having charge $q = e$ come out of an accelerator rom A with kinetc energy $K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}$…(1). Here, m is mass of proton and $\vec{v} =$ is the velocity. The proton beam bends while passing through magnetic field $\vec{B} \perp \vec{v}$.  As per Lorentz's Force Law, which is $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{F} = q\vec{v} \times B\hat{k} \Rightarrow \vec{F}_m = qvB\sin\theta\,(-\hat{r})$ .Here, angle of $\vec{B}$ w.r.t. $\vec{v}$ is given to be $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1$ and unit radial vector $\hat{r}$ as shown in the figure. Accordingly, $\vec{F}_m = qvB(-\hat{r})$…(2), centripetal force responsible for circular motion of the proton. Magnetic force at A is in a direction perpendicular to the velocity of ejection by accelerator. It will not will change the speed and circular trajectory of protons is shown in the figure. The proton while describing circular it will experience it will experience centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(3). . In state of uniform motion, i.e. equilibrium, $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r}$. It leas to $B = \frac{mv^2}{qvr} \Rightarrow B = \frac{mv}{qr}$…(4).

	Using the available data, $B=\frac{m\left(\sqrt{\frac{2K}{m}}\right)}{er} \Rightarrow B=\frac{\sqrt{2mK}}{er}\ldots(5)$. It is given that magnitude of the magnetic field is such that it just misses a target placed at distance l from the accelerator. From the the geometry of of the circular path of proton in magnetic field it isseen that $r=\frac{l}{2}$, accordingly (5) gets transformed to $B=\frac{\sqrt{2mK}}{e\frac{l}{2}} \Rightarrow B=\frac{2\sqrt{2mK}}{el} \Rightarrow B=\frac{\sqrt{8mK}}{el}$ **is the answer.** **N.B.:** This problem needs careful analysis of motion of the charged particle. Accordingly, the target along the accelerator will always be missed. It must be along a line perpendicular to the initial velocity at A, the instant of ejection from the accelerator. Rest of the problem is application of electromagnetic forces and mechanics of circular motion.
Q-13	A particle of mass m and positive charge q moving wth a uniform velocity v, enters a magnetic field B as shown in the figure.  (a) Find the radius of the circular arc it descrbes in the magnetic field. (b) Find the angle subtended by the arc at the center. (c) How long does the particle stay inside the magnetic field? (d) Solve the three parts of the above problem if charge q on the particle is negative.
A-13	(a) $\frac{mv}{qB}$ (b) $\pi-2\theta$ (c) $\frac{m}{qB}(\pi-2\theta)$ (d) $\frac{mv}{qB}, \pi+2\theta, \frac{m}{qB}(\pi+2\theta)$
I-13	This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass mass m and charge q moving with a velocity $\vec{v}=v\hat{v}$ enters a magnetic field   $\vec{B}=B(-\hat{k})$ as shown in the figure. It is seen that velocity vector $\vec{v} \perp \vec{B}$. Therefore, magnetic force experienced by the particle $\vec{F}_m=q\vec{v}\times\vec{B}$. It leads to $\vec{F}_m=qvB(\hat{n})\ldots.(1)$. Here, $\hat{n}$ is perpendicular to both the vectors $\vec{v}$ and $\vec{B}$. This is a condition of circular motion where $\vec{F}_m$ acts as centripetal force such that $\hat{n}\rightarrow(-\hat{r})\ldots(2)$, and $\hat{r}$ is the radius vector of the circular path. With this pre-analysis of the system, each part is being solved separately – **Part (a):** The particle during circular motion would experience a centrifugal force $\vec{F}_C=\frac{mv^2}{r}\hat{r}\ldots(3)$. During the uniform circular motion forces are in equilibrium. Thus, combining (1), (2) and (3) it leads to $\vec{F}_m+\vec{F}_C=0 \Rightarrow qvB(-\hat{r})+\frac{mv^2}{r}\hat{r}=0 \Rightarrow qvB=\frac{mv^2}{r} \Rightarrow \boldsymbol{r}=\frac{\boldsymbol{mv}}{\boldsymbol{qB}}$**…(4), is answer of part (a)** **Part (b):** Let A is the point at which charged particle is entering the magnetic field at an angle θ and after taking a circular path of radius r, having center at C, it exits the magnetic field at B, as shown in the figure $\angle ACB=\alpha=(\pi-2\theta)\ldots(5)$. Thus, geometrically the arc AB subtends an angle $(\boldsymbol{\pi-2\theta})$**) at its center C is the answer of part (b).**  **Part (c):** Time spend by the particle in the magnetic field which is performing uniform circular motion with speed v is $t=\frac{rength\ of\ the\ arc}{speed\ of\ the\ particle}\ldots(6)$. Length of the arc AB $l_{AB}=r\alpha\ldots(7)$. Thus combining (4)…(7) we have $t=\frac{r(\pi-2\theta)}{v} \Rightarrow t=\frac{\frac{mv}{qB}(\pi-2\theta)}{v} \Rightarrow t=\frac{m(\pi-2\theta)}{qB}\ldots(8)$.Thus, **answer of part (c) is** $\boldsymbol{\frac{m(\pi-2\theta)}{qB}}$

Part (d): In this case charge of the particle is $(-q)$. Therefore, analysis would be on the lines in part (a)..(c) except in all the equations. Therefore direction of magnetic force would reverse leading to the trajectory of the path of the particle as shown in the figure.

Since, magnitude of the magnetic force and counterbalancing centrifugal force remain unchanged. Hence, radius of the path of uniform circular motion would remain same as $\boldsymbol{r = \frac{mv}{qB}}$. Further, parajectory of the particle is major arc of the circle and hence geometrically angle formed by the major arc at the centre C of the trajectory is $\boldsymbol{\pi + 2\theta}$. As regards speed of the particle as weel as radius of the circular path remain unchanged. Hence, **time taken by the particle to come out of the magnetic field is** $\boldsymbol{\frac{m(\pi+2\theta)}{qB}}$.

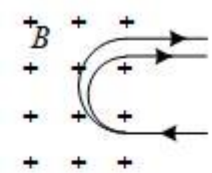
Thus, **answer of the part (d) is** $\boldsymbol{\frac{mv}{qB}, \pi + 2\theta, \frac{m(\pi+2\theta)}{qB}}$

Thus, **answers are (a)** $\boldsymbol{\frac{mv}{qB}}$ **(b)** $\boldsymbol{\pi - 2\theta}$ **(c)** $\boldsymbol{\frac{m}{qB}(\pi - 2\theta)}$ **(d)** $\boldsymbol{\frac{mv}{qB}, \pi + 2\theta, \frac{m}{qB}(\pi + 2\theta)}$.

N.B.: (1) In such in part (d) analytical equations remain same as in part (a)…(c), except for the change in charge from $q \rightarrow (-q)$. Accordingly there is change in trajectory of the charged particle. Thus affecting change in geometry, wherever necessary, symmetry of equations can be utilized to abridge the answer, unless Part (d) is an independent question.

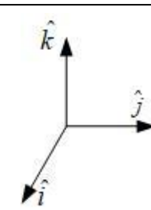
(2) This problem involves uniform speed of particle along a circular trajectory. Hence, $t = \frac{rength\ of\ the\ arc}{speed\ of\ the\ particle}$ is correct. However, solving it as, on the lines of projectile motion $t = \frac{Displacement\ AB}{Component\ of\ velocity\ vector\ along\ AB}$ would be incorrect.

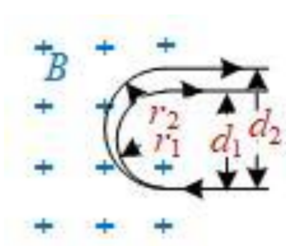
Q-14 A narrow beam of singly-charged carbon ions, moving with a constant velocity of 6.0×10^4 m/s, is sent perpndicularly in a rectangular region having a uniform magnetic field $B = 0.5$ T as shown in the figure. It is found that two beams emerge from the field in the backward direction., the separation from the incident beam being 3.0 cm and 3.5 cm. Identify the carbon isotopes present in the ion beam. Take the mass of the ions $= A(1.6 \times 10^{-27})$ kg where A is the mass number.



A-14 ^{12}C and ^{14}C

I-14 This problem involves 3D vectors and hence unit vectors are shown in the figure. Given is a particle of mass mass m and charge q moving with a velocity $\vec{v} = v(-\hat{j}) = 6.0 \times 10^4(-\hat{j})$ enters a magnetic field $\vec{B} = B(-\hat{k}) = 0.5(-\hat{k})$T as shown in the figure. It is seen that velocity vector $\vec{v} \perp \vec{B}$. Therefore, magnetic force experienced by the particle, as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = q\left(6.0 \times 10^4(-\hat{j})\right) \times \left(0.5(-\hat{k})\right) \Rightarrow \vec{F}_m = q(3.0 \times 10^4)\hat{n}$…(1).

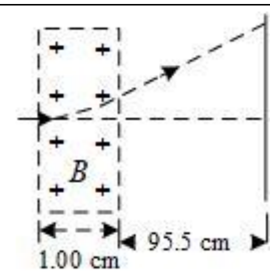




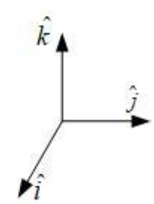
It is case of circular motion where $\hat{n}$ is perpendicular to both vectors $\vec{v}$ and $\vec{B}$ and $\hat{n} = (-\hat{r})$…(2) Moreover, ions are singly charged, yet initial direction of deflection as given is along $(-\hat{\imath})$ and hence charge on ions must be negative. It leads to $\vec{F}_m = qvB(-\hat{r})$….(3).

The particle during circular motion would experience a centrifugal force $\vec{F}_C = \frac{mv^2}{r}\hat{r}$…(4). During the uniform circular motion forces are in equilibrium. Thus, combining (1), (2) and (3) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow qvB(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$…(5), is radius of the circular trajectory of the ions.

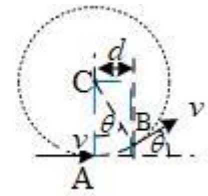
	We are given two isotopes whose masses are m_1 and m_2 and, therefore, ratio of their radii is $\frac{r_1}{r_2}=\frac{\frac{m_1 v}{qB}}{\frac{m_2 v}{qB}}$. It leads to $\frac{r_1}{r_2}=\frac{m_1}{m_2}$…(6). It is given that ions emerge out of the magnetic field in backward direction and hence seperation of incident and emergent ions is diameter of the circular trajectory of the ions where $d = 2r$ and likewise atomic mass is $m = A(1.6\times 10^{-27})$, here A is atomic number of the atom. Accordingly, (6) is transformed into $\frac{d_1}{d_2}=\frac{m_1}{m_2}\Rightarrow\frac{2\times r_1}{2\times r_2}=\frac{A_1\times(1.6\times10^{-27})}{A_2\times(1.6\times10^{-27})}\Rightarrow\frac{d_1}{d_2}=\frac{Z_1}{Z_2}$…(7). Usimg the given data in (7), $\frac{3}{3.5}=\frac{m_1}{m_2}\Rightarrow\frac{6}{7}=\frac{m_1}{m_2}$…(8). Considering the atomic structure ions having mass $m_1$is ^{12}C and the other ion is $m_1$is ^{14}C whose atomic numbers are 12 and 14 respectively. Thus, answer is ^{12}C and ^{14}C. **N.B.:** It is seen from the illustration that all the given data is notional and is not required when solving the problem algebiacally. It, however, requires understanding of atomic numbers for iosotopes. is seen from the illustration that all the given data is notional and is nome of it is required when solving the problem algebiacally. It is, therefore, advised that numerical solution should not be attempted unless it is essential. It saves time and brings in accuracy of results. It, however, requires understanding of atomic numbers for iosotopes.
Q-15	A narrow beam of singly charged potassium ions of kinetic energy 32 keV is injected into a region of width 1.00 cm having a magnetic field $B = 0.500$ T as shown in the figure. The ions re-collected at a screen 95,5 cm away from the field region. If the beam contains isotopes of atomic weights 39 and 41, find the separation between the points where these isotopes strike the screen. Take the mass of a potassium ion $m = A(1.6\times10^{-27})$ kg where A is the mass number.



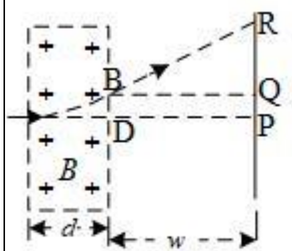
A-15	0.75mm
I-15	This problem involves 3D vectors and hence unit vectors are shown in the figure.Given are single charged potessioum ions having kinetic energy $K = 32\times10^3$eV. For reference energy $1eV = 1.6\times10^{-19}$J, thus $K=(32\times10^3)(1.6\times10^{-19})$ J. Such ions are injected along $(\hat{j})$ into a magnetic field $B = 0.500(-\hat{k})$ T …(1) of width $d = 1.00\times10^{-2}$m. Let the ion acquires a velocity $\vec{v}=v\hat{j}$ then $K=\frac{1}{2}mv^2$…(2). Here m is the mass of the ion. Accordingly, $\frac{1}{2}mv^2=K\Rightarrow v=\sqrt{\frac{2K}{m}}$…(2),. Hence, velocity vector of the ion. Is $\vec{v}=v\hat{j}$…(3). It is seen that velocity vector $\vec{v}\perp\vec{B}$. This ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F}_m=q\vec{v}\times\vec{B}\Rightarrow\vec{F}_m=e\left(\sqrt{\frac{2K}{m}}\hat{j}\right)\times B\hat{k}\Rightarrow\vec{F}_m=\left(eB\sqrt{\frac{2K}{m}}\right)\hat{\imath}$. This is a case of circular motion of ion where $\vec{v}\perp\vec{F}_m$, accordingly ceptripetal acceleration would acts along $(-\hat{r})=\hat{\imath}$. This concludes to $\vec{F}_m=\left(eB\sqrt{\frac{2K}{m}}\right)(-\hat{r})$…(4). While the ions describe circular motion wouldit experience a centrifugal force $\vec{F}_C=\frac{mv^2}{r}\hat{r}$…(5). We know that mass number of potassium is A, therefore, $m = A\times(1.6\times10^{-27})$kg.



During the uniform circular motion the two forces $\vec{F}_m$ and $\vec{F}_C$ are in equilibrium. Thus, combining (4) and (5) it leads to $\vec{F}_m + \vec{F}_C = 0 \Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right)(-\hat{r}) + \frac{mv^2}{r}\hat{r} = 0 \Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right) = \frac{m\left(\sqrt{\frac{2K}{m}}\right)^2}{r}$. It leads to $\Rightarrow \left(eB\sqrt{\frac{2K}{m}}\right) = \frac{2K}{r} \Rightarrow r = \frac{\sqrt{2K\times m}}{eB}$…(6).



Circular trajectory of the ion through narrow magnetic field is shown in the figure. Ion during motion inside magnetic field is deflected through an angle θ, which trigonometrically is $\sin\theta = \frac{d}{r}$…(7), as shown in the figure. Taking $\theta \ll \Rightarrow \sin\theta \to \theta \Rightarrow \theta = \frac{d}{r}$…(8)



Outside the magnetic field it reaches the screen along BR as shown in the figure. Therefore, for ions of kotassium with $A_1 = 39$ and $A_2 = 41$, their striking points R on the screen at a distance wf from the magnetic field would be, above point P, at height $h = \text{RP} - \text{QP} \Rightarrow \frac{h}{w} = \tan\theta$…(9). With the approximation at (8), we have $\tan\theta \to \theta = \frac{d}{r}$. Accordingly, $\frac{h}{w} = \frac{d}{r} \Rightarrow h = \frac{d\times w}{r}$ …(11). This for the two ions is h_1 and h_2.

Accordingly, seperation between the two ions striking on the screen is $\Delta h = |h_1 - h_2|$. It further solves into $\Delta h = \left|\frac{d\times w}{r_1} - \frac{d\times w}{r_2}\right| \Rightarrow \Delta h = (d\times w)\left|\frac{1}{r_1} - \frac{1}{r_2}\right|$ ...(12).

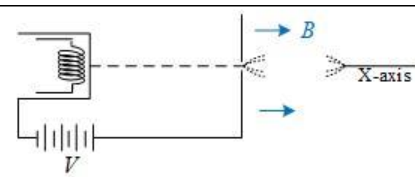
Combining (6) and (12), $\Delta h = (d\times w)\left|\frac{1}{\frac{\sqrt{2K\times\left(A_1\times(1.6\times10^{-27})\right)}}{e\times B}} - \frac{1}{\frac{\sqrt{2K\times\left(A_2\times(1.6\times10^{-27})\right)}}{e\times B}}\right|$. It further solves into $\Delta h = \frac{(d\times w)\times(e\times B)}{\sqrt{2K\times(1.6\times10^{-27})}}\left|\frac{1}{\sqrt{A_1}} - \frac{1}{\sqrt{A_2}}\right|$.

Using the available data, $\Delta h = \frac{\left((1.00\times10^{-2})\times(95.5\times10^{-2})\right)\times\left((1.6\times10^{-19})\times(0.500)\right)}{\sqrt{2\times\left((32\times10^{3})\times(1.6\times10^{-19})\right)\times(1.6\times10^{-27})}}\left|\frac{1}{\sqrt{39}} - \frac{1}{\sqrt{41}}\right|$

$$\Rightarrow \Delta h = (1.89\times10^{-1})\times(3.954\times10^{-3}) \Rightarrow \boldsymbol{\Delta h = 0.75\ \text{mm is the answer.}}$$

N.B. This illustration uses approximation in arriving at values of θ and h, and has been accordingly brought out in it.

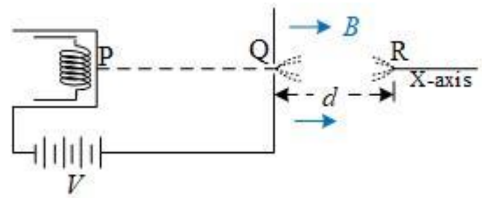
Q-16 Electrons emitted with negligible speed from an electron gun are accelerated thriugh a potential difference V along the X-axis. These electrons emerge from a narrow hole into a uniorm magnetic field B directed along this axis. However, some of the electrons emerging from the hole make slightly divergent angle as shown in the figure. Show that these paraxial electrons are refocussed on the X-axis at a distance $\sqrt{\frac{8mV}{eB^2}}$.



A-16 -

I-16

Let, mass of electron is m, carries a charge $q = -e$ moves is emitted with a velocity $u \ll$ and is accelerated by a electric potential difference V along a seperation $\vec{d} = d\hat{\imath}$ as shown in the figure.

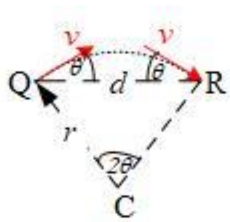


Kinetic energy gained by the electron during travel from P to Q would be $K = eV = \frac{1}{2}mv^2$. Hence, $v = \sqrt{\frac{2eV}{m}}$…(1).

Further, there is magnetic field alongmagnetic field is $\vec{B} = B\hat{\imath}$ …(2).

These aligned electrons ion in the magnetic field would experience a force as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B} \Rightarrow \vec{F}_m = cvB\sin\theta\,\hat{n} \Rightarrow \vec{F}_m = e(v\hat{v}) \times B\hat{\imath} \Rightarrow \vec{F}_m = F_m\hat{n} \Rightarrow F_m = evB$…(3).

Non-divergent electrons would not experience magnetic force since both $\hat{v} = \hat{\imath}$ and hence from (3) we have $\vec{F}_m = e(v\hat{\imath}) \times B\hat{\imath} \Rightarrow \vec{F}_m = evB(\hat{\imath} \times \hat{\imath}) = 0$, since $\hat{\imath} \times \hat{\imath} = 0$. Accordingly, time taken by the electron along QR would be $t = \frac{d}{v} \Rightarrow t = \sqrt{\frac{md^2}{2eV}}$…(4).



But, electrons diverted by any angle say θ would experience, as per (3), magnetic force $\vec{F}_m = evB\sin\theta\,\hat{n}$ ... (4). along $(\hat{n})$ is perpendicular to both the vectors $\hat{v}$ and $\hat{\imath}$ and hence eventually $\hat{v} \perp \hat{n}$ is a valid case of circular motion. Hence, divergent electron would experience a uniform circular motion of radius r such that $F_c = \frac{mv^2}{r}$…(5). In case of uniform circular motion of radius r, there would be equilibrium of forces. Hence, $F_m = F_c \Rightarrow evB\sin\theta = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{eB\sin\theta}$…(6).

Electrons emerging at Q have same kinetic energy K and hence same speed v. Yet, possibility of slight diversion by an angle θ in the merging electrons is not ruled out. Electro-mechanics of such electrons reaching R reveal a geometrical symmetry as shown in the figure. Yet, dependence of $F_m \propto \sin\theta$ in (3) and radius of curvature of the arc described by the electrons $r \propto \frac{1}{\sin\theta}$ in (6) makes $d = QR$ independent of θ. It is discussed in footnote to the illustration. Accordingly length of the chord would be $QR = 2r\sin\theta \Rightarrow QR = 2\left(\frac{m\left(\sqrt{\frac{2eV}{m}}\right)}{eB\sin\theta}\right)\sin\theta$. It leads to $QR = \sqrt{\frac{8mV}{eB^2}}$, **is the answer.**

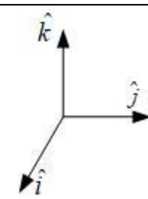
N.B.: Electro-mechanical analysis involved in the problem is of intresting relevance, and is being discussed. **C**hange in momentum during time Δt taken by deviated electron to describeng motion along the arc QR is $\Delta p = mv\big((\sin\theta\ \hat{\jmath} + \cos\theta\ \hat{\imath}) - (-\sin\theta\ \hat{\jmath} + \cos\theta\ \hat{\imath})\big) \Rightarrow \Delta p = 2mv\sin\theta\ \hat{\jmath}$. Let then as per mechanics $\frac{\Delta p}{\Delta t} = F_m \Rightarrow \frac{2mv}{\Delta t}\sin\theta = evB\sin\theta \Rightarrow \Delta t = \frac{2m}{eB}$ …(7). It is seen that time taken to reach R by all electrons is independent of angle of diversion

Electron, is describing circular motion with speed v with a radius r and time period of the circular motion is $T = \frac{\textit{Perimeter of the circular trajectory}}{\textit{Speed of the electron}} \Rightarrow T = \frac{2\pi r}{v}$. Using (6) it leads to $T = \frac{2\pi\left(\frac{mv}{eB\sin\theta}\right)}{v} \Rightarrow T = \frac{2\pi m}{eB\sin\theta}$. It leads to length of the arc QR is $\frac{\Delta t}{T} = \frac{PQ_{arc}}{2\pi r} \Rightarrow QR_{arc} = \left(\frac{\Delta t}{T}\right)2\pi r \Rightarrow QR_{arc} = 2\pi\left(\frac{\frac{2m}{eB}}{\frac{2\pi m}{eB\sin\theta}}\right)\left(\frac{mv}{eB\sin\theta}\right)$. It solves into

	$QR_{arc} = \left(\frac{2mv}{eB}\right)$. Accordingly length of the chord would be $QR = 2r\sin\theta \Rightarrow QR = 2\left(\frac{m\left(\sqrt{\frac{2eV}{m}}\right)}{eB\sin\theta}\right)\sin\theta$. It leads to $QR = \sqrt{\frac{8mV}{eB^2}}$.
Q-17	A proton projected in a magnetic field of 0.020 T travels along a helical path of radius 5.0 cm and pitch 20 cm. Find the component of the velocity of proton along and perpendicular to the magnetic field. Take mass of the proton $= 1.6 \times 10^{-27}$.
A-17	$\mathbf{6.4 \times 10^2}$**m/s and** $\mathbf{1.0 \times 10^3}$ **m/s**

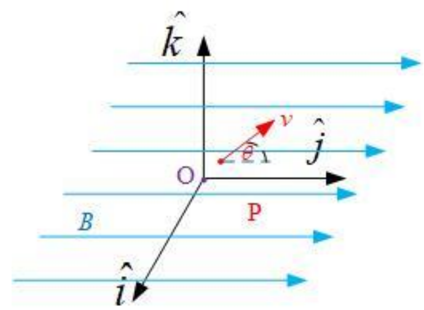
I-17

For convenience of analysis 3D unit vectorss are shown in the figure. Given is a proton having mass $m = 1.6 \times 10^{-27}$kg and charge $q = 1.6 \times 10^{-19}$ C. It is projected in magnetic field $\vec{B} = B\hat{b} \Rightarrow \vec{B} = 0.020\hat{\jmath}$. Trajectory of the proton is helical with radius $r = 5.0 \times 10^{-2}$m and pitch $\lambda = 2.0 \times 10^{-1}$m.



Let, the proton is projcted with a velocity v making an angle θ with the magnetic field such that $\vec{v} = v_j\hat{\jmath} + v_k\hat{k} \Rightarrow \vec{v} = v\cos\theta\,\hat{\jmath} + v\sin\theta\,\hat{k}$…(1).

A moving charge would experience magnetic force as per Lorentz's Force Law, $\vec{F}_m = q\vec{v} \times \vec{B}$…(2). Here, $\hat{n} \to \hat{k}$ is vector perpendicular to plane of vectors $\vec{v}$ and $\vec{B}$. The equation (2) essentially requires charge to be in motion and therefore $v \neq 0$. Therefore, combining (1) and (2) we have $\vec{F}_m = qvB\left(\cos\theta\,\hat{\jmath} + \sin\theta\,\hat{k}\right) \times \hat{\jmath} \Rightarrow \vec{F}_m = qvB\left(\cos\theta\,\hat{\jmath} \times \hat{\jmath} + \sin\theta\,\hat{k} \times \hat{\jmath}\right)$. It leads to $\vec{F}_m = qvB\left(0 + \sin\theta\,(-\hat{\imath})\right)$…(3).



Analysis of (3) reveals that it is only velocity component $v_k\hat{k} = v\sin\theta\,\hat{k}$ which affects creates a $\vec{F}_m \perp v_k\hat{k}$, force motion of the charge and thus qualifies for a uniform circular motion. While, $v_j\hat{\jmath}$ being along $\vec{B}$ does not affect motion of the charge. Thus, it leads to resultant motion of charge which is superimposition of translation motion on circular motion, eventually it is of helix form which a radius and a pitch.

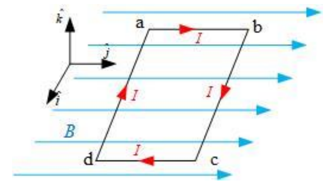
Therefore, both circular motion is being analyzed to determine radius of the helx. This radius will be used to determine time period of circular motion, which in turn will help to determine pitch of helix using translational motion of the particle.

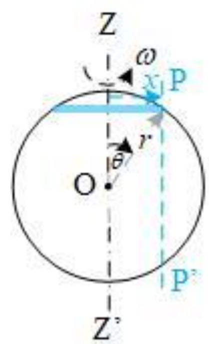
Circular Motion: Magnetic force $\vec{F}_m$is the cause of uniform circular motion, while centrifugal force $\vec{F}_c = \frac{mv^2}{r}\hat{r}$…(4) is the reaction which creates an equilibrium of forces such that $\vec{F}_m + \vec{F}_c = 0$…(5), as per Newton's Third Law of Motion. Thus, in accordance with Newton's First Law of Motion its speed of rotation v and radius r of the circular trajectory remains constant.

Comining (3), (4) and (5), $qvB\sin\theta\,(-\hat{\imath}) + \frac{m(v\sin\theta)^2}{r}\hat{r} = 0 \Rightarrow v\sin\theta = \frac{(5.0\times10^{-2})(1.6\times10^{-19})(0.020)}{1.6\times10^{-27}}$…(6). It solves into $v\sin\theta =$ $\mathbf{1.0 \times 10^3}$**m/s is the velocity o the proton perpendicular the the magntic field, is one part of the solution.**

The charge is since describing circular motion of radius r with a uniform speed v, its time period is $T = \frac{2\pi r}{v\sin\theta}$…(7). Hence, using available data $T = \frac{2\pi\times(5.0\times10^{-2})}{(1.0\times10^3)} \Rightarrow T = 3.14 \times 10^{-4}$s…(8).

Translational Motion: While describing circular motion the charged particle continues to traverse with translational velocity $v_j\hat{\jmath} = v\cos\theta\,\hat{\jmath}$ and traverses along $\hat{\jmath}$ a distance $\lambda = v_j \times T \Rightarrow \lambda = v\cos\theta\,T$.

	Therefore, velocity of charge along the magnetic field is $v\cos\theta = \frac{\lambda}{T}$. Using the available data we have $v\cos\theta = \frac{2.0\times10^{-1}}{3.14\times10^{-4}} \Rightarrow v\cos\theta = 6.4\times10^{2}$m/s. Thus, **answer is $\mathbf{1.0\times10^{3}}$ m/s and $\mathbf{6.4\times10^{2}}$m/s.**
Q-18	A rectangular coil of 100 turns has length 5 cm and width 4 cm. it is placed with its plane parallel to a uniform magnetic field and a current of 2 A is sent through the coil. Find the magnitude of the magnetic field B, if the torque acting on the coil is 0.2 Nm.
A-18	0.5 T
I-18	Given system is shown in the figure with 3D-unit vectors The coil has 100 torns of in rectangular shape having length $l = 5\times10^{-2}$m and width $w = 4\times10^{-2}$. The coil carries a current $I = 2$ A. It is required to finf magnetic field B whereby the the coil exeriences a net torque $\Gamma_{net} = 0.2$ Nm.  A current carrying conductor placed in magnetic field, as [er Lorentz's Force Law experiences magnetic force $\vec{F} = I\vec{l}\times\vec{B} \Rightarrow \vec{F} = IlB\sin\theta\,\hat{n}$…(1). Here, θ is angle of $\vec{B}$ w.r.t. $\vec{l}$ and $\hat{n}$ is unit direxction vector perpendicular to the plane containing vectors $\vec{l}$ and $\vec{B}$. It is to be noted that length vector is taken along the direction of the current in it. Accordingly for two opposite sides, $\vec{l}_{ab} = l_{ab}\hat{j}$ while $\vec{l}_{cd} = l_{cd}(-\hat{j})$. Same principle is used for other two sides of the rectangular coil. Given is since a coil of turns $n = 100$ in shape abcd and hence to take n times the forces on each side of rectangle abcd, using (1). Accordingly forces – **Side ab:** $\vec{F}_{ab} = I(l_{ab}\hat{j})\times(B\hat{j}) \Rightarrow \vec{F}_{ab} = Il_{ab}B(\hat{j}\times\hat{j}) = 0$, since $\hat{j}\times\hat{j} = 0$. **Side bc:** $\vec{F}_{bc} = I(l_{bc}\hat{\imath})\times(B\hat{j}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{\imath}\times\hat{j}) \Rightarrow \vec{F}_{bc} = Il_{bc}B(\hat{k})$.. **Side cd:** $\vec{F}_{cd} = I(l_{cd}(-\hat{j}))\times(B\hat{j}) \Rightarrow \vec{F}_{cd} = -Il_{cd}B(\hat{j}\times\hat{j}) = 0$, since $\hat{j}\times\hat{j} = 0$. **Side da:** $\vec{F}_{da} = I(l_{da}(-\hat{\imath}))\times(B\hat{j}) \Rightarrow \vec{F}_{da} = -Il_{da}B(\hat{\imath}\times\hat{j}) \Rightarrow \vec{F}_{da} = Il_{bc}B(-\hat{k})$. It is to be noted that while $\vec{F}_{ab}$ and $\vec{F}_{cd}$ the other two forces $\vec{F}_{bc}$ and $\vec{F}_{da}$ are- (a) equal in magnitude $F_{bc} = F_{da} = Il_{bc}B$, (b) opposite directions (c) seperated by a distance equal to width of the coil w. (d) The above three are a valid case of a torque on the turn of the coil. Accordingly, torque on the couple $\vec{\Gamma} = \vec{w}\times\vec{F} \Rightarrow \vec{\Gamma} = \vec{l}_{ab}\times\vec{F}_{bc} \Rightarrow \vec{\Gamma} = (l_{ab}\hat{j})\times(Il_{bc}B\hat{k})$. It further solves to $\vec{\Gamma} = Il_{ab}l_{bc}B(\hat{j}\times\hat{k}) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$...(2). Here, $A = Il_{ab}l_{bc}$…(3), is the area of turns of the coil. Given that the coil has n turns of same area and hence net torque would be $\vec{\Gamma}_{net} = n\vec{\Gamma} \Rightarrow \vec{\Gamma}_{net} = nIAB\hat{\imath} \Rightarrow B = \frac{\Gamma_{net}}{nIA}$…(4). Using the available data, $B = \frac{0.2}{100\times2\times((5\times10^{-2})(4\times10^{-2}))} \Rightarrow \boldsymbol{B = 0.5}$**T is the answer.** **N.B.:** Equation (2) can also be written as $\vec{\Gamma} = IAB\hat{\imath} = I\vec{A}\times\vec{B}$ …(5). In this case $\vec{A} = A\hat{a}$ where unit vector $\hat{a}$ is along perpendicular to the area A such that for observer if current in the loop is anti-clockwise then $\hat{a}$ is (+)ve i.e. towards the observer. Whereas, if current in the loop is clockwise then $\hat{a}$ is (-)ve i.e. away from the observer. In the instant case, as shown in the figure, when we observe the coil from the top current is clockwise and hence $\hat{a} = (-\hat{k})$. Accordingly, $\vec{\Gamma} = IA(-\hat{k})\times(B\hat{j}) \Rightarrow \vec{\Gamma} = IAB(-\hat{k})\times(\hat{j}) \Rightarrow \vec{\Gamma} = IAB\hat{\imath}$…(6). It is seen that conclusion at (6) conforms to (2), used to solvethe problem.

	Thus, ***equation (6) alongwith direction vector of area can be use as takeaway for handling problems involving torque experienced by a current carrying coil placed in uniform magnetic field.***
Q-19	Consider a solid sphere of radius r and mass m which has a charge q distributed uniformly over its volume. The sphere is rotated about its diameter with an angular speed ω. Show that the magnetic moment μ and angular momentum L of the plate are related as $\mu = \frac{q}{2m}L$.
A-19	-
I-19	Given is a non-conducting solid sphere of radius r and mass m carries a uniformly distributed charge q. The ring is rotated with an angular velocity ω about its axis Z-Z' as shown in the figure. Thus charge density in the sphere is $\rho = \frac{q}{\frac{4}{3}\pi r^3} \Rightarrow \rho = \frac{3q}{4\pi r^3}$...(1).  Since current $i = \frac{\Delta Q}{\Delta t}$...(2). Since, angular speed of the disc is $\omega = 2\pi N \Rightarrow N = \frac{\omega}{2\pi}$, here N is number revolutions per second. Therefore, time of one revolution of the disc is $T = \frac{1}{N} \Rightarrow T = \frac{2\pi}{\omega}$...(3). In time $\Delta t = T$ the complete sphere passes through a line PP' parallel to ZZ', axis of rotation and hence charge on the ring passes through the OP under consideration is $\Delta Q = q$. Accordingly, equivalent current in the ring is $i = \frac{q}{\frac{2\pi}{\omega}} \Rightarrow i = \frac{q\omega}{2\pi}$...(4). We know that magnetic moment of a coil is $\vec{\mu} = i\vec{A}$...(5). Therefore, current due to distributed charge needs to be analyzed by decomposing the disc into elemental cylinders of radius $0 < x < r$ of radial thickness $\Delta x \to 0$. Here, $x = r\sin\theta \Rightarrow \Delta x = r\cos\theta\,\Delta\theta$...(6). Accordingly let us take an elemental cylinder of radius $0 < x < r$ of radial thickness $\Delta x \to 0$ and height $h = 2r\cos\theta$. Therefore, charge on the ring $\Delta q = (2\pi x\Delta x \times h)\rho \Rightarrow \Delta q = (2\pi x\Delta x \times 2r\cos\theta)\left(\frac{q}{\frac{4}{3}\pi r^3}\right)$. It leads to $\Delta q = \frac{3q}{r^2}\cos\theta\,(r\sin\theta)(r\cos\theta\,\Delta\theta) \Rightarrow \Delta q = 3q\cos^2\theta\sin\theta\,\Delta\theta$...(7). Let, $\cos\theta = u \Rightarrow -\sin\theta\,\Delta\theta = \Delta u$ ...(8). Combining (7) and (8), we get $\Delta q = -3qu^2\Delta u$...(9). Combining (2), (3) and (9), current in the cylinder is $\Delta i = \frac{\Delta q}{T} \Rightarrow \Delta i = \frac{-3qu^2\Delta u}{\frac{2\pi}{\omega}} \Rightarrow \Delta i = -\frac{3q\omega}{2\pi}u^2\Delta u$...(10). Therefore, as per (5) magnet moment of the elemental cylinder is $\Delta\mu = \Delta iA \Rightarrow \Delta\mu = \left(-\frac{3q\omega}{2\pi}u^2\Delta u\right)(\pi x^2)$ ...(11). Combining (6) in (11), $\Delta\mu = \left(-\frac{3q\omega}{2}u^2\Delta u\right)(r^2\sin^2\theta) \Rightarrow \Delta\mu = \left(-\frac{3q\omega r^2}{2}u^2\Delta u\right)(1-u^2)$. It further solves into $\Delta\mu = \frac{3q\omega r^2}{2}(u^4 - u^2\Delta u)\Delta u$...(12). Integrating (12), $\mu = \frac{3q\omega r^2}{2}\left(\frac{u^5}{5} - \frac{u^3}{3}\right)$...(13). Reverting back to the variable $u \to \cos\theta$ and limits of $\theta = 0$ to $\theta = \frac{\pi}{2}$ we get $\mu = -\frac{3q\omega r^2}{2}\left[\frac{\cos^5\theta}{5} - \frac{\cos^3\theta}{3}\right]_0^{\frac{\pi}{2}}$. It, further, reduces to $\mu = \frac{3q\omega r^2}{2}\left(-\left(\frac{1}{5} - \frac{1}{3}\right)\right) \Rightarrow \mu = \frac{3q\omega r^2}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{3q\omega r^2}{2}\left(\frac{2}{15}\right) \Rightarrow \mu = \frac{q\omega r^2}{5}$...(14) Angular momentum of a ring is $L = I\omega$, here moment of inertia of a sphere about its axis is $I = \frac{2mr^2}{5}$. Therefore, $L = \frac{2mr^2\omega}{5} \Rightarrow \omega r^2 = \frac{5L}{2m}$...(15). Combining (14) and (15) we have $\mu = \left(\frac{q}{5}\right)\left(\frac{5L}{2m}\right) \Rightarrow \boldsymbol{\mu = \frac{q}{2m}L}$, **hence proved.** **N.B.:** Non-conducting material has a property that at normal conditions charges remain at place and do not flow. Therefore, current is produced by charges distributed on non-conducting geometry only when the geometry

	changes its position. In this case geometry of non-conducting is a disc and displacement of charges is created by angular motion of the disc about its axis. This principle can be applied to any geometry of non-conducting material.

N.B.: Complete set of Questions, Answers and their Illustrations are posted on the web along with Preamble

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