

LET US DO SOME PROBLEMS: XXXXI

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The Board Examinations of Class XII students are starting soon. Some important Questions are hereby selected for their practice and understanding the level of the Questions.

QUESTIONS

1. If $y = e^{\sin^{-1}x}$ and $z = e^{-\cos^{-1}x}$, prove that $\frac{dy}{dz} = e^{\frac{\pi}{2}}$.

2. Prove that the function $f(x) = x^3 - 6x^2 + 12x + 5$ is increasing on R .

3. Using properties of determinants prove that:

$$\begin{vmatrix} x & x(x^2 + 1) & x + 1 \\ y & y(y^2 + 1) & y + 1 \\ z & x(z^2 + 1) & z + 1 \end{vmatrix} = (x - y)(y - z)(z - x)(x + y + z)$$

4. If $\sec^{-1}x = \operatorname{cosec}^{-1}x$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

5. Show that the function $f(x) = |x - 4|$, $x \in R$ is continuous, but not differentiable at $x = 4$.

6. If $f: R \rightarrow R$, $f(x) = x^3$ and $g: R \rightarrow R$, $g(x) = 2x^2 + 1$, and R is the set of real numbers, then find $f \circ g(x)$ and $g \circ f(x)$.

Ans. $8x^6 + 12x^4 + 6x^2 + 1$, $2x^6 + 1$

7. Solve: $\sin(2 \tan^{-1}x) = 1$.

Ans. $x = 1$

8. Using determinants, find the values of k , if the area of triangle with vertices $(-2, 0)$, $(0, 4)$ and $(0, k)$ is 4 square units.

Ans. $k = 8$

9. Evaluate: $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$

Ans. $\tan x - x + C$

10. Using L Hospital's Rule, evaluate: $\lim_{x \rightarrow 0} \frac{8^x - 4^x}{4x}$

Ans. $\frac{1}{4} \log 2$

11. Two balls are drawn from an urn containing 3 white, 5 red and 2 black balls, one by one without replacement. What is the probability that at least one ball is red?

Ans. $\frac{7}{9}$

12. If events A and B are independent, such that $P(A) = \frac{3}{5}$, $P(B) = \frac{2}{3}$, find $P(A \cup B)$.

Ans. $\frac{1}{3}$

13. If $f: A \rightarrow A$ and $A = \mathbb{R} - \left\{\frac{8}{5}\right\}$, show that the function $f(x) = \frac{8x+3}{5x-8}$ is one-one onto. Hence, find f^{-1} .

Ans. $f^{-1}(y) = \frac{8y+3}{5y-8}$ for all $y \in \mathbb{R} - \left\{\frac{8}{5}\right\}$

14. Solve for x : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

Ans. $x = \pm \frac{1}{\sqrt{2}}$

15. A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground, at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 5 m away from the wall?

Ans. $-\frac{5}{6}$ m/sec

16. Evaluate: $\int \frac{x(1+x^2)}{1+x^4} dx$

Ans. $\frac{1}{2} \tan^{-1} x^2 + \frac{1}{4} \log|1+x^4| + C$

17. Evaluate: $\int_{-6}^3 |x+3| dx$

Ans. $\frac{45}{2}$

18. Solve the differential equation: $\frac{dy}{dx} = \frac{x+y+2}{2(x+y)-1}$

Ans. $6y - 3x - 5 \log|3x + 3y + 1| = C$

19. Bag A contains 4 white balls and 3 black balls, while Bag B contains 3 white balls and 5

black balls. Two balls are drawn from Bag A and placed in Bag B. Then, what is the probability of drawing a white ball from Bag B?

Ans. $\frac{29}{70}$

20. Solve the following system of linear equations using matrix method:

$$\begin{aligned}\frac{1}{x} + \frac{1}{y} + \frac{1}{z} &= 9 \\ \frac{2}{x} + \frac{5}{y} + \frac{7}{z} &= 52 \\ \frac{2}{x} + \frac{1}{y} - \frac{1}{z} &= 0\end{aligned}$$

Ans. $x = 1, y = \frac{1}{3}, z = \frac{1}{5}$

21. The volume of a closed rectangular metal box with a square base is 4096cm^3 . The cost of polishing the outer surface of the box is Rs. 4 per cm^2 . Find the dimensions of the box for the minimum cost of polishing it.

Ans. $16\text{cm} \times 16\text{cm} \times 16\text{cm}$

22. Find the point on the straight line $2x+3y=6$, which is closest to the origin.

Ans. $\left(\frac{12}{13}, \frac{18}{13}\right)$

23. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

Ans. $\pi \left(\frac{\pi}{2} - 1\right)$

24. Given three identical boxes A, B and C. Box A contains two gold and one silver coin. Box B contains one gold and two silver coins and Box C contains three silver coins. A person chooses a box at random and takes out a coin. If the coin is of silver, find the probability that it has been drawn from the Box which has the remaining two coins also of silver.

Ans. 0.5

25. If \vec{a} and \vec{b} are perpendicular vectors, $|\vec{a} + \vec{b}| = 13$, and $|\vec{a}| = 5$. Find the value of $|\vec{b}|$.

Ans. 12

26. If \vec{a} and \vec{b} are non-collinear vectors, find the value of x such that the vectors $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$ are collinear.

Ans. $x = \frac{1}{4}$

27. Find the equation of the plane passing through the intersection of the planes $2x+2y-3z-7=0$, and $2x+5y+3z-9=0$ such that the intercepts made by the resulting plane on the x -axis and the z -axis are equal.

Ans. $12x+27y+12z=52$

28. The following results were obtained with respect to two variables x and y :

$\Sigma x=15$, $\Sigma y=25$, $\Sigma xy=83$, $\Sigma x^2=55$, $\Sigma y^2=135$, and $n=5$.

Find the regression coefficient b_{xy} . Also find the regression equation of x on y .

Ans. $b_{xy} = \frac{4}{5}$ $5x-4y+5=0$