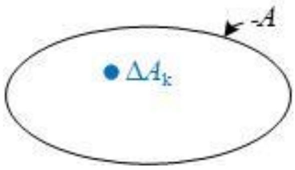
Electromagnetism: Electromagnetic Induction [Part 3- Set 4(a)]

Typical Questions with Illustrations

Q-01 A metallic loop is placed in a non-uniform magnetic field. Will an emf be induced in the loop?

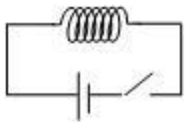
I-01 Given that magnetic field is non-uniform in which a metallic loop of area $\vec{A}$ is placed. Such a system is shown in the figure considering k^{th} of area $\Delta\vec{A}_k = \Delta A\hat{a}_k$, in the area under consideration. Let us take magnetic field intercepting the area $\Delta\vec{A}_k$ is $\vec{B}_k = B_k\hat{b}_k$, Thus, flux linking the element $\Delta\phi_k = \Delta\vec{A}_k.\vec{B}_k$…(1). The dot-product in (1) is $\Delta\phi_k = \Delta A_k B_k \cos\theta_k \Rightarrow \phi = \oint (B_k \cos\theta_k)\, dA_k$…(2), here θ_k is angle between vectors $\Delta\vec{A}_k$ and $\vec{B}_k$. But, as per Faraday's Law Magnetic induction, emf induced in the loop is $E = \frac{d}{dt}\phi$…(3). Given that the metallic loop is placed in non-uniform magnetic field which time invariant, though not mentioned specifically. It implies that there is no change in position of coil and hence the flux linking the coil in (2) is a constant. Accordingly, (3) leads to $E = 0$. Hence, no emf will be induced in the loop, is the answer.



N.B: Analyzing a mathematically in the beginning might appear to be tedious and lengthy. Yet, with little practice it complements to objective clarity in application of basic concepts. Thereby, it wipes of any kind of ambiguity, a necessity for scientific learning.

Q-02 An inductor is connected to a battery through a switch. Explain why the emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when switch is closed?

I-02 Given system is shown in the figure. Let the inductance of the inductor is L and battery emf is E. A real circuit has some electrical resistance. Though the circuit does not show resistance but the real resistance is taken to be R. Thus the given circuit is an L-R circuit.



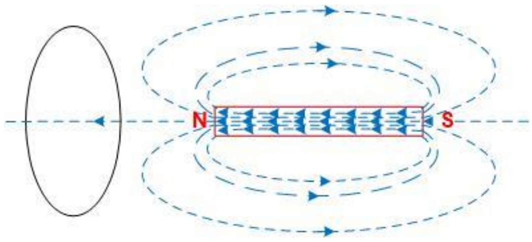
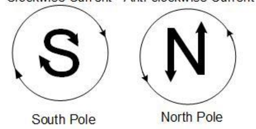
In such a it is essential to illustrate three stages of the circuit as under –

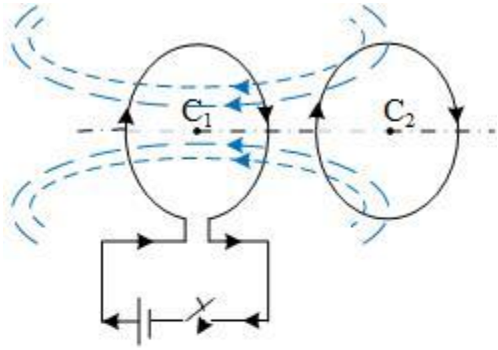
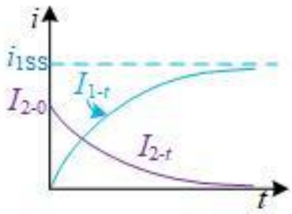
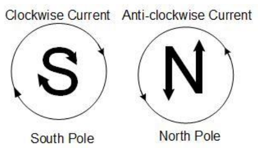
1- **Switching ON:** when current from increases from $i_o = 0$ to an instantaneous value at any time t is $i_t = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \Rightarrow i_t = \frac{E}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$…(1). Here, $\tau = \frac{L}{R}$ is called time constant of the circuit. This is result of solution of voltage equation of circuit $E - L\frac{di}{dt} = Ri$…(2), as per Kirchhoff's Loop Law, together with definition of inductance $L = \frac{\psi}{i}$…(3), property of inductor, derived using Laws of Electromagnetism. Graphical representation of current in (1) is called exponential rise of current with time. Here, ψ is the flux linking the inductor when current i flows through it. The (2) is a linear differential equation.

   Initial condition $t = 0$, in (1), leads to current $i_0 = 0$, while at $t = \tau \Rightarrow i_\tau = \frac{E}{R}\left(1 - e^{-\frac{\tau}{\tau}}\right) \Rightarrow i_\tau = \frac{E}{R}(1 - e^{-1})$. It leads to $i_\tau = \frac{E}{R}(1 - 0.36788) \Rightarrow i_\tau = 0.63212\left(\frac{E}{R}\right) \Rightarrow i_\tau \approx 0.63\left(\frac{E}{R}\right)$. Therefore, using (2), at time $t = 0$ voltage across inductor is $e_{LC} = L\frac{di}{dt} = E - Ri_0 \Rightarrow e_{LC} = E$. Further in time constant τ it decays to $e_{L\tau} = L\frac{di}{dt} = E - Ri_\tau \Rightarrow e_{L\tau} = E - R \times 0.63\left(\frac{E}{R}\right) \Rightarrow e_{L\tau} = E(1 - 0.63) \Rightarrow e_{L\tau} = 0.37E$

2- **Steady State:** when $t \to \infty$, the exponential factor in (1) tends to $\frac{1}{e^{\frac{L}{R}t}} \to 0$. Accordingly, (1) leads to $i_{SS} = \frac{E}{R}$…(4). It is the maximum value. It is seen that i_{SS} is independent of inductance in the circuit and controlled by E and R . Moreover, (2) reveals that emf induced in the inductor while closing the switch is $e_{\text{LC}} = L\frac{di}{dt} \Rightarrow e_{\text{LC}} = E - Ri \Rightarrow e_{\text{LC}} < E$…(5)

3- **Switching OFF:** Let switch in the circuit is opened when circuit reaches steady state, role of battery and resistance disappears and, therefore, there is no emf in the circuit to drive the current. Yet, inductance comes into full bloom.

	Accordingly, equation (1) changes to $-L\frac{\Delta i}{\Delta t} = e$…(6). Here, $\Delta i = 0 - i_{\text{SS}} \Rightarrow \Delta i = -i_{\text{SS}}$ which occurs abruptly i.e. $\Delta t \rightarrow 0$. Thus, (6) leads to $e_{\text{LO}} = -L\frac{(-i_{\text{SS}})}{0} \Rightarrow e_{\text{LO}} \rightarrow \infty$, a theoretical limit. In reality, spark occurs across the switch to discharge magnetic energy in the inductor to cause decay of current to Zero. From the above illustrations it is clear that emf induced in the inductor when the switch is opened is $e_{\text{LO}} \rightarrow \infty$; while when the switch is closed $e_{LC} = E$. It implies that $e_{\text{LO}} \gg e_{\text{LC}}$. As stated in the problem. **The reason behind it is the rate of change of flus in the inductor** $\frac{\Delta\psi}{\Delta t} = \boldsymbol{L}\frac{\Delta i}{\Delta t}$ above.
Q-03	The coil of a moving-coil galvanometer keeps on oscillating for a long time if it is deflected and released. If ends of the coil are connected together, the oscillations stop at once. Explain.
I-03	Moving coil galvanometer has a coil, it gets deflected from mean position when current is passed through it. The coils acts as $R - L$ circuit. Magnetic energy gained by the coil due to current is counter balanced by restraining force, a design feature of the instrument, while resistance of the coil R acts like damper Any change in current establishes oscillation of coil, and it continues until it is dissipated in the damper. When, ends of coil are open, the coil-circuit is open. Thereby, no way to dissipate the magnetic energy of the coil. In such a condition exchange of energy in the system between the coil and restraining-force continues for a long. But, when ends of coil are connected together, the coil circuit is completed and dissipation of energy starts occurring to damp the oscillation quickly. Thus reason is that **in open coil there is no damping and oscillation continues, while in coil with it ends connected damping occurs to quickly stop the oscillation.**
Q-04	A short magnet is moved along the axis of a conducting loop. Show that the loop repels the magnet if the magnet is approaching the loop and attracts the magnet if it is going away from the loop.
I-04	 Conducting loop is a closed circuit having certain area $\vec{A} = A\hat{a}$. A short magnet produces non uniform magnetic field $\vec{B}_k = B_k\hat{b}_k$ around it as shown in the figure. Here, suffix k is the k^{th} elemental area of the coil such that $\vec{A} = \sum \Delta\vec{A}_k$. As a result flux linking the elemental area of the loop is $\Delta\phi_k = \Delta\vec{A}_k.\vec{B}_k \Rightarrow \Delta\phi_k = \Delta A_k B_k \cos\theta_k$. Accordingly, flux linking the loop is oil at any instant is $\phi = \oint(B_k \cos\theta_k)\, dA_k$…(1). Conceptually for component of flux along axis of the loop the angle between $\hat{a}_k$ and $\hat{b}_k$ unit vectors is $\theta_k = \pi$ and hence $\phi = -\oint(B_k \cos\theta_k)\, dA_k$ As per Faraday's Law of Magnetic Induction emf induced in the loop is $E = -\frac{d}{dt}\phi$…(2) When magnet is moved towards the loop, more of the magnetic lines of force from North to South pole would intercept the area of the coil, taking different elemental area. Therefore, for magnet under motion flux linking the loop is $\phi = -f(t)$. Therefore, for magnet moving towards the loop $\frac{d}{dt}\phi > 0$ i.e. (+)ve and hence $E = -\frac{d}{dt}\big(-f(t)\big) \Rightarrow E = \frac{d}{dt}f(t)$ is (+ve). Thus, as per Ohms Law it will drive current in the loop $I = \frac{E}{R} \Rightarrow I = \frac{\frac{d}{dt}f(t)}{R} \Rightarrow I = \frac{1}{R}\left(\frac{d}{dt}f(t)\right)$…(3) This (+)ve current in the loop is in anti-clockwise direction tending to cause North pole on the loop facing the magnet, as shown in the figure; it is in accordance with Bio-Savart-Ampere's Law. Eventually it will tend to oppose the increase in flux linking the coil due to motion of bar magnet towards to the loop and is in accordance with the Lenz's Law which states that '*current induced in a loop/coil by flux changes are in such a direction that magnetic flux caused by induced current opposes it*'. 

	Therefore, as per principle of magnetic forces two like poles would repel each other so also **the loop and bar magnet moving towards each other would repel each other**. Applying the same principles, conversely, **loop and bar magnets moving away from each other would attract each other**. **N.B.:** This problem is a good example of integration of laws of electromagnetism.
Q-05	Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. (a) Will there be a current induced in the other loop? (b) If yes, when does the current start and when does it end? (c) Do the loops attract each other or do they repel?
I-05	Given system of two loops, Loop-1(L1) and Loop 2 (L2) having centers C1 and C2 and resistance of the loops R_1 and R_2 respectively, are shown in the figure. L2 is closed while the L1 closes with a battery of emf E and a switch. Initially the switch is open. At $t = 0$ when switch closed, it will build a time-varying current $i_{1t} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$ in the L1 starting from $i_{10} = 0 \rightarrow i_{1SS} = \frac{E}{R}$...(1).  Current in L1 will produce time-varying magnetic field B_t in space around it as per Bio-Savart-Ampere's Law. Part of the magnetic field flux, due to L1, would create a flux linkage ϕ_{t_21} with L2. Thus, **answer to the part (a), is Yes.**  Graphical representation of current in (1) is called exponential rise of current with time as shown in the figure. But, **current i_{2-t} induced in L2 is maximum at $t = 0$, in accordance with $\frac{d}{dt}\phi_{t\_21}$ i.e. emf induced in L2 and proportionate current in it. But it decays with time**, **is answer of part (b).** This ϕ_{t_21}would induce a current i_{2t} in L2, which opposes increase in ϕ_{t_21} caused by current i_{1t} in L1, as per (1). Thus, i_{2t} would produce flux ϕ_{t_2} in L2. As per Lenz's Law $\phi_{t\_2} = -\phi_{t\_21}$. It eventually leads to like poles would appears on the faces of both loop facing each other. It implies that if face of L1 towards L2 has South pole then face of L2 towards L1 will also have South pole and vice versa. Thus, **the loops would repel each other is the answer, is answer of part (c).**  Thus, **answers are (a) Yes, (b) Start with closing of switch and stops when current in L1 becomes i_{1SS} and (c) repel each other.**

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