LET US DO SOME PROBLEMS – XXXXII

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Some questions are selected here for the Aspirants who are preparing for the coming IIT-JEE Mains. These questions will help in understanding the standard of the questions.

QUESTIONS

Q1. If *n* is the number of the solutions of the equation $2cosx\left(4sin\left(\frac{\pi}{4}+x\right)sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$ and *S* is the sum of all these solutions, then the ordered pair (*n*, *S*) is

(a)(3, 5π/3) (b) (2, 8π/9)
(c) (3, 13π/9) (d) (2, 2π/3)

Ans.(c)

Q2. $cos^{-1}(\cos(-5))+sin^{-1}(\sin(6))-tan^{-1}(\tan(12))$ is equal to

(a)3π-11 (b) 3π+1
(c) 4π-11 (d) 4π-9

Ans.(c)

Q3. Let $a_1,a_2,\ldots,a_{21}$ be an AP such that. If the sum of this AP is 189, then a_6a_{16} is

(a)57 (b)36
(c)48 (d)72

Ans. (d)

Q4. The range of the function $f(x)=\log_{\sqrt{5}}\left(3+cos\left(\frac{3\pi}{4}+x\right)+cos\left(\frac{\pi}{4}+x\right)+cos\left(\frac{\pi}{4}-x\right)-cos\left(\frac{3\pi}{4}-x\right)\right)$ is

(a)[-2,2] (b)$\left[\frac{1}{\sqrt{5}},\sqrt{5}\right]$
(c)$\left(0,\sqrt{5}\right)$ (d)[0,2]

Ans. (d)

Q5. Let the acute angle bisector of the two planes *x-2y-2z=-1* and *2x-3y-6z=-1* be the plane *P*. then which of the following points lies on *P*?

(a)(0,2,-4) (b)(4,0,-2)
(c)(-2.0,-1/2) (d)(3,1,-1/2)

Ans.(c)

Q6. The number of pairs (*a*,*b*) of real numbers, such that whenever is α a root of the equation $x^2+ax+b=0$, α^2-2 is also a root of this equation, is:

(a)6 (b)4
(c)8 (d)2

Ans.None of these

Q7. Let *f(x)* be a polynomial of degree 3 such that $f(k)=-\frac{2}{k}$ for *k=2,3,4,5*. Then the value of *52 - 10 f(10)* is equal to

Ans.26

Q8. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, then the greatest coefficient in the expansion is

Ans. 924

Q9. A fair dice is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability P(X ≥5| x>2) is

(a)5/6 (b)25/36
(c)125/216 (d)11/36

Ans.(b)

Q10. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations $x+y+z=5$, $x+2y+3z=\mu$, and $x+3y+\lambda z=1$ is constructed. If ρ is the probability that the system has a unique solution, *q* is the probability that the system has no solution, then

(a)*p=5/6, q=5/36*
(b)*p=1/6, q=1/36*

(c)$p=1/6, q=5/36$
(d)$p=5/6, q=1/36$
Ans.(a)

Q11. The locus of the mid points of the chords of the hyperbola $x^2-y^2=4$, which touch the parabola $y^2=8x$, is
(a)$y^3(x-2)=x^2$ (b)$y^2(x-2)=x^3$
(c)$x^3(x-2)=y^2$ (d)$x^2(x-2)=y^3$
Ans.(b)

Q12. The domain of the function$cosec^{-1}\left(\frac{1+x}{x}\right)$ is
(a)$(-1, -1/2]\cup(0,\infty)$ (b)$[-1/2, \infty)-\{0\}$
(c)$[-1/2,0) \cup[1, \infty)$ (d)$(-1/2, \infty)-\{0\}$
Ans.(b)

Q13. If the value of the integral $\int_0^5 \frac{x=[x]}{e^{x-[x]}}dx = \alpha e^{-1}+\beta$, where $\alpha, \beta \in R$, $5\alpha+6\beta=0$ and [x] denotes the greatest interer less than or equal to x, then the value of $(\alpha+\beta)^2$ is equal to
(a)16 (b)100
(c)25 (d)36
Ans.(c)

Q14. Let $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 1 & 0 & 0\end{pmatrix}$. Then $A^{2025}-A^{2020}$ is equal to
(a)A^5 (b)A^6
(c)A^5-A (d)A^6-A
Ans.(d)

Q15. $\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1(x+4}\right)$ is equal to
(a)9/44 (b)5/24
(c)7/36 (d)1/5
Ans.(a)

Q16. Let $\lambda \neq 0$ be in *R*. If α and β are the roots of the equation $x^2-x+2\lambda=0$; α, and γ are the roots of the equation $3x^2-10x+27\lambda=0$, then $(\beta\gamma/\lambda)$ is equal to
Ans.18

Q17. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and thay all are multiple of 11 , is
Ans.7744

Q18. Let the mean and variance fo four numbers 3,7, *x* and *y* (*x*>*y*) be 5 and 10 respectively. Then the mean of four numbers $3+2x$, $7+2y$, $x+y$ and $x-y$ is
Ans.12