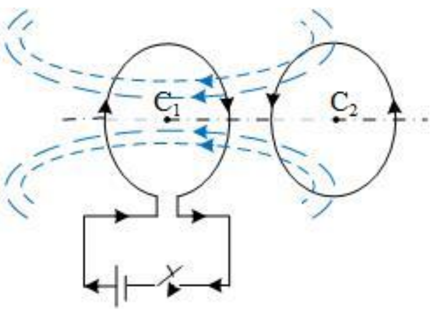
Electromagnetism: Electromagnetic Induction [Part 3- Set 4(a-02)]

Typical Questions with Illustrations

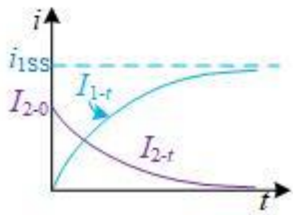
Q-01 Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it.

(a) Will there be a current induced in the other loop?
(b) If yes, when does the current start and when does it end?
(c) Do the loops attract each other or do they repel?

I-01 Given system of two loops, Loop-1(L1) and Loop 2 (L2) having centers C1 and C2 and resistance of the loops R_1 and R_2 respectively, are shown in the figure. L2 is closed while the L1 closes with a battery of emf E and a switch. Initially the switch is open. At $t = 0$ when switch closed, it will build a time-varying current $i_{1t} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$…(1), in the L1 starting from $i_{10} = 0 \rightarrow i_{1SS} = \frac{E}{R}$…(2).

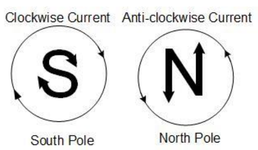


Current in L1 will produce time-varying magnetic field B_t in space around it as per Bio-Savart-Ampere's Law. Part of the magnetic field flux, due to L1, would create a flux linkage say ϕ_{t_21} with L2. Thus, **answer to the part (a), is Yes.**



Graphical representation of current in (1) is called exponential rise of current with time as shown in the figure. But, **current i_{2-t} induced in L2 is maximum at $t = 0$, in accordance with $\frac{d}{dt}\phi_{t\_21}$ i.e. emf induced in L2 and proportionate current in it. But it decays with time**, **is answer of part (b).**

This ϕ_{t_21}would induce a current i_{2t} in L2, which opposes increase in ϕ_{t_21} caused by current i_{1t} in L1, as per (1). Thus, i_{2t} would produce flux ϕ_{t_2} in L2. As per Lenz's Law $\phi_{t\_2} = -\phi_{t\_21}$. It eventually leads to like poles that woiuld appears on the faces of both loop facing each other. It implies that if face of L1 towards L2 has South pole then face of L2 towards L1 will also have South pole and vice versa. Thus, **the loops would repel each other is the answer, is answer of part (c).**

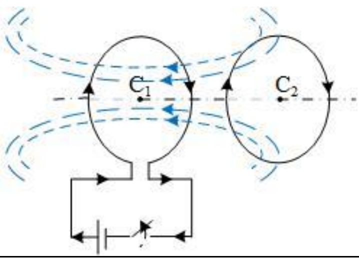


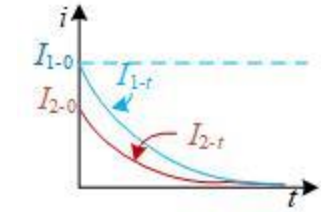
Thus, **answers are (a) Yes, (b) Start with closing of switch and stops when current in L1 becomes i_{1SS} and (c) repel each other.**

Q-02 Two circular loops are placed coaxially but separated by a distance. A switch connected to the battery is closed till steady state is reached. The battery is suddenly disconnected by opening the switch.

(a) Is a current induced in the other loop?
(b) If yes, when does it start and when does it end?
(c) Do the loops attract each other or repel?

I-02 Given system of two loops, Loop-1(L1) and Loop 2 (L2) having centers C1 and C2 and resistance of the loops R_1 and R_2 respectively, are shown in the figure. While, L2 is a closed loop, battery of emf E in loop L1 is initially closed till steady state current in the loop is reached. At $t = 0$, when switch is opened, current in the loop L1 would start decaying $i_{1t} = \frac{E}{R}e^{-\frac{R}{L}t}$ starting from $i_{10} = \frac{E}{R} \rightarrow i_{1SS} = 0$…(1).





At $t = 0$ current in L1 will produce a constant flux ϕ_1 and a part of it say ϕ_{21}, which is also constant would link L2.

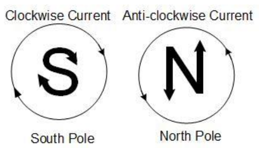
When switch is opened current in L1 tends to decay and so also flux would ϕ_{1t}, and it becomes time variant. Since, ϕ_{1t} is decaying $\frac{d}{dt}\phi_{1_t} \rightarrow (-)$ve.

Thus, time varying flux linking L2 as $\phi_{21_t} \rightarrow (-)$ve, and induce emf in L2 in accordance with Faraday's Law of Magnetic Induction. This emf in turn would induce current in L2 and consequent flux. Hence, **answer to part (a) is Yes**.

Direction flux as per Lenz's Law would be to oppose cause of changes; it implies that when rate of change of flux linking L1 is $\frac{d}{dt}\phi_{1\_t} \rightarrow (-)$ ve, then rate of change of flux linking L2 will be $\frac{d}{dt}\phi_{2_t} \rightarrow (+)$ve. This ϕ_{2_t} is flux linking L2, and a part of it ϕ_{12_t}would link L1. Thus, increase in ϕ_{2_t} and in turn ϕ_{12_t} would tend to compensate decrease in ϕ_{1_t}.

The above discussions about change of flux are linked to change of current, and so also direction of flux to direction of current as per Biot-Savart's Law. Combining the effect of Farrday's Law coupled with Biot-Savarts Law, the current L2 **starts with opening of the switch and decays with decay of the current in L1 is the answer of Part (b).** This effect is depicted in $i - t$ graph, above.

Direction polarity of magnetic field is related to direction of current in the loop, as shown in the figure. When



L1, as seen from L2 carries clockwise current, polarity of L1 w.r.t L2 is It is seen that, in first figure, direction of current is clockwise and in torn polarity of the coil seen from L2 is South. Flux produced by L2, which is trying to compensate decrease of flux in L1, must be of opposite polarity. Accordingly, direction of current produced in L2 must be same as that in L1.

It is analogues to two parallel conductors carrying unidirectional current. Such conductors experience force of attraction. Hence the loops would **experience attraction, answer of part (c)**.

Thus answers are –

Part (a) -Yes

Part (b) - Start with opening of the switch, and stops when current in L1 becomes $Zero$,

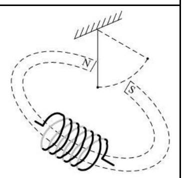
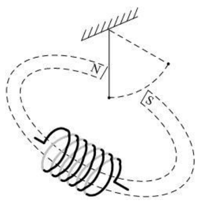
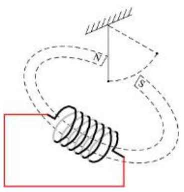
Part (c) - The loops would attract each other is the answer.

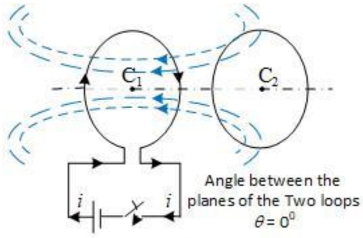
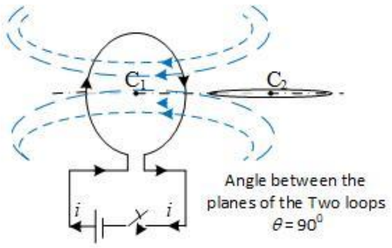
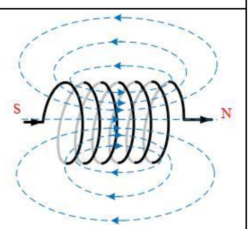
Q-03 If the magnetic field outside a copper box is suddenly changed, what happen to the magnetic field inside the box? Such low-resistivity metals are used to form enclosures which shield objects inside them against varying magnetic fields.

I-03 Magnetic field outside a copper box intercepts surface area of the box. The Box can be conceived to be composed of concentric rings encircling the oncoming magnetic field. When magnetic field changes $\left(A = \frac{d\phi}{dt}\right)$…(1), suddenly. It produces **(a)** emf in the rings (Faraday's First Law of Electromagnetic induction), **(b)** the magnitude of the emf proportional to the rate of change of flux (Faraday's Second Law of Electromagnetic induction, say $E \propto A$), **(c)** The emf E induced in the rings, by changing magnetic field, will establish current inversely proportional to the resistance of the rings as per Ohm's Law. Accordingly, $I \propto \frac{A}{R}$, **(d)** Current I induced in the rings will produce magnetic field ϕ' along the axis of the rings which is proportional to the current I in the ring (Biot-Savart's Law). These currents are called eddy currents. Accordingly, $\phi' \propto I \Rightarrow \phi' \propto \frac{A}{R}$ **(e)** Direction and nature of change of flux induced ϕ' would oppose the change in magnetic field A as per Lenz's Law. It leads to $\frac{d\phi'}{dt} \propto \left(-\frac{1}{R} \times \frac{d\phi}{dt}\right) \Rightarrow \frac{d\phi''}{dt} = \left(-\frac{K}{R} \times \frac{d\phi}{dt}\right)$, here K is proportionality constant.

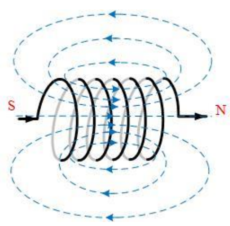
Thus, effective flux changes $\frac{d\phi''}{dt}$ in the space occupied by the metal box is $\frac{d\phi''}{dt} = \frac{d\phi}{dt} + \frac{d\phi'}{dt} \Rightarrow \frac{d\phi''}{dt} = \frac{d\phi}{dt} - \frac{K}{R} \times \frac{d\phi}{dt}$. It leads to $\frac{d\phi''}{dt} = \left(1 - \frac{K}{R}\right)\frac{d\phi}{dt}$…(2).

	A close observation of (2) reveals that for copper which has low resistivity, being a good conductor, $R \ll$. Therefore, the cancelling magnetic field ϕ'' is large. Accordingly, changing magnetic field is cancelled by the enclosure. Thus, **inside the enclosure changes in magnetic field are not experienced, is the answer.**
Q-04	Metallic (non-ferromagnetic) and non-metallic particles in a solid waste may be separated as follows. The waste is allowed to slide down an incline over permanent magnets. The metallic particles slow down as compared to the nonmetallic ones and hence are separated. Discuss of eddy currents in the process.
I-04	Metallic (non-ferromagnetic) and non-metallic particles in a solid waste when allowed to slide down an incline, over a magnetic field, an electromagnetic phenomenon, called eddy currents, occurs and is explained in steps-wise manner - (a) During siding on incline magnetic field intercepting particles changes due to change of distance from permanent magnet and surface area of the particles intercepting magnetic field. Change in surface area is due to change of angle of the surface of particle w.r.t. magnetic field; this change occurs to readjustment of position of the particle during slide. (b) Changes at (a) above lead to change of magnetic flux w.r.t. and can be expressed as $A = \frac{d\phi}{dt}$…(1). (c) Magnetic field is intercepted by surface of metallic (non-magnetic) particles. This surface can be conceived to be composed of concentric rings encircling the oncoming magnetic field. (d) When magnetic field changes at (1) produce emf in the rings (Faraday's First Law of Electromagnetic induction), (e) The magnitude of the induced emf is proportional to the rate of change of flux (Faraday's Second Law of Electromagnetic induction, say $E \propto A$). (f) The emf E , induced in the rings, will establish current inversely proportional to the resistance of the rings as per Ohm's Law. Accordingly, $I \propto \frac{A}{R}$. (g) Current I induced in the rings will produce magnetic field ϕ' along the axis of the rings which is proportional to the current I in the ring (Biot-Savart's Law). These currents are called eddy currents. Accordingly, $\phi' \propto I \Rightarrow \phi' \propto \frac{A}{R}$ (h) Direction and nature of change of flux induced ϕ' would oppose the change in magnetic field A as per Lenz's Law. It leads to $\frac{d\phi'}{dt} \propto \left(-\frac{1}{R} \times \frac{d\phi}{dt}\right) \Rightarrow \frac{d\phi''}{dt} = \left(-\frac{K}{R} \times \frac{d\phi}{dt}\right)$, here K is proportionality constant. (i) Thus, effective flux changes $\frac{d\phi''}{dt}$ in the space occupied by the metal box is $\frac{d\phi''}{dt} = \frac{d\phi}{dt} + \frac{d\phi'}{dt} \Rightarrow \frac{d\phi''}{dt} = \frac{d\phi}{dt} - \frac{K}{R} \times \frac{d\phi}{dt}$. It leads to $\frac{d\phi''}{dt} = \left(1 - \frac{K}{R}\right)\frac{d\phi}{dt}$…(2). A close observation of the above process which culminates into (2) reveals that linking **for metallic (non-magnetic particles) electromagnetic coupling is strong since $R \ll$.** It leads to electromagnetic retardation of such particles which overcomes gravitational acceleration such particles while the waste slides down a slope. While, **non-metallic particles** also undergo the same process, **but the electromagnetic coupling in (2) is low due the fact that these particles having $R \gg$.**
Q-05	A pivoted aluminum bar falls much more slowly through a region containing magnetic field than a similar bar of an insulating material. Explain.
I-05	Given are two bars, one of aluminum and the other is of insulated; they are pivoted. They fall through a region where magnetic field exists. It is a free fall under gravity. Therefore, velocity of fall for both would though be the same, yet the downward velocities would change with time (in accordance with the first equation of motion) i.e. $v = -gt$…(1). Accordingly, the rate at which the rods would intercept magnetic field, $A = \frac{d\phi}{dt} \Rightarrow A \propto \frac{d(gt)}{dt} \Rightarrow A \propto g$…(2). These two rods would experience an electromagnetic induction and is explained in steps-wise manner -

	(a) Surface of the rods can be conceived to be composed of concentric rings encircling the oncoming magnetic field. (b) When magnetic field changes as per (2), it produces an emf in the rings (Faraday's First Law of Electromagnetic induction), (c) The magnitude of the induced emf is proportional to the rate of change of flux (Faraday's Second Law of Electromagnetic induction, say $E \propto A \Rightarrow E \propto g$…(3) (d) The emf E , induced in the rings, will establish current inversely proportional to the resistance of the rings as per Ohm's Law. Accordingly, $I \propto \frac{A}{R} \;\Rightarrow\; I \propto \frac{g}{R}$ …(4) (e) Current I induced in the rings will produce magnetic field ϕ' along the axis of the rings which is proportional to the current I in the ring (Biot-Savart's Law). These currents are called eddy currents. Accordingly, $\phi' \propto I \Rightarrow \phi' \propto \frac{g}{R}$ …(5). (f) Direction and nature of change of flux induced ϕ' would oppose the change in magnetic field A as per Lenz's Law. It leads to $\frac{d\phi'}{dt} \propto (-)\frac{g}{R} \Rightarrow \frac{d\phi''}{dt} = \left(-\frac{kg}{R}\right)$,..^) (g) Thus, effective flux changes $\frac{d\phi''}{dt}$ in the space occupied by the metal box is $\frac{d\phi''}{dt} = \frac{d\phi}{dt} + \frac{d\phi'}{dt} \Rightarrow \frac{d\phi''}{dt} = g - \frac{kg}{R}$. It leads to $\frac{d\phi''}{dt} = \left(1 - \frac{K}{R}\right) g$…(7). A close observation of the above process which culminates into (7) reveals that **electromagnetic coupling in aluminum rod is strong since $R \ll$.** It leads to electromagnetic retardation of such particles which overcomes gravitational acceleration experienced by it. While, insulating rod also undergo the same process, **but the electromagnetic coupling in (7) is not there. It is attributed to the fact that resistance of insulating material $R \to \infty$.**
Q-06	A metallic bob A oscillates through the space between the poles of an electromagnet, as shown in the figure. The oscillations are more quickly damped when the circuit is on, as compared to the case when the circuit is off. Explain 
I-06	The oscillating bob experiences a simple harmonic motion $v = V\cos\theta$…(1). Here, θ a is angular displacement of the bob from mean position. Thus, the velocity of the bob is maximum when bob is in the mean position i.e. $\theta = 0$.  The bob is given to be metallic and hence when the bob passes through the poles it intercepts magnetic field between the poles at velocity which is changing with angular displacement as per (1). This motion would induce eddy currents on the surface of the bob and thus loos of kinetic energy of the bob in the form of heat energy. This loss of energy would create a marginal damping of the oscillating bob when the circuit is off. It implies that ends of the coil of the electromagnet are open.  Given that when circuit is on, implying that ends of the coil of the electromagnet are short-circuited as shown in the figure, the eddy currents induced in bob would interact with magnetic field of the electromagnet. This interaction would be counterproductive to magnetic field of the electromagnet in accordance with the Lenz's Law. Thus, changes in the magnetic field of the electromagnet would produce an emf E in coil as per Faraday's Laws of electromagnetic induction. This induced emf will cause flow of current E through the closed circuit as per Ohm's Law. Every circuits has some resistance say r. Thus, it will convert electrical energy U into heat energy $H = \frac{E^2}{r}$ as per Joule's Law. The electrical energy U is derived from kinetic energy of the bob. Thus, **when the circuit is closed there is additional conversion of kinetic energy of the bob and it leads to quick damping.**
Q-07	Two circular loops are placed with their centers separated by a fixed distance. How would you orient the loops to have –

	(a) the largest mutual inductance (b) the smallest mutual inductance
I-07	Given are two circular loops L_1 and L_2 separated by a distance with their centers C_1 and C_2 respectively. When current say i flows through the loop L_1, applying Bio-Savart's Law magnetic flux ϕ_1 would be linking the loop can be determined. The flux $\phi_1 \propto i$...(1). The proportionality constant in (1) is related to geometry of the loop.  Fig 1: Planes of the Two loops Parallel to each other The loop L_2 is since separated the flux produced by L_1 that would link the loop L_2 is ϕ_{21}; it is dependent upon - (a) geometry of L_2 and its angular position w.r.t. L_1. Accordingly, $\phi_{21} \propto \phi_1 \cos\theta$…(2), here θ is the angle between the planes of the two loops.  Fig 2: Planes of the Two loops Perpendicular to each other Combining (1) and (2), $\phi_{21} \propto i\cos\theta$…(3). Therefore, emf induced in L_2, as per Faraday's Law of Electromagnetic Induction, due to change of current in L_1 is $e_{21} \propto (-)\frac{d\phi_{21}}{dt}$ …(4). Combining (3) and (4), $e_{21} \propto \left(\cos\theta \frac{di}{dt}\right)$…(5). This equation can be expressed as $e_{21} = M\frac{di}{dt}$ …(6). In this, $M = K\cos\theta$…(7), is called mutual inductance of the two loops. In M coefficient $\cos\theta$ is implicit. Given systems are shown in Fig 1 and Fig 2 having, having surfaces of the loop parallel and perpendicular to each other. in Fig. 2, where the loops are parallel the angle $\theta = 0^0 \Rightarrow \cos\theta = 1$…(8). Accordingly, combining (7) and (8) , we have $M_1 = K$…(9). Whereas, in Fig. 2, where the loops are perpendicular to each other $\theta = 90^0 \Rightarrow \cos\theta = 0$ …(10). Therefore, combining (7) and (10), $M_2 = 0$…(11). Observing (9) and (11), **mutual inductance of the loops is largest when loops are oriented parallel, answer of part (a)**; and **smallest when loops are oriented perpendicular to each other, answer of part (b).**
Q-08	Consider the self-inductance per unit length of a solenoid at the center and that near its ends. Which of the two is greater?
I-08	Electric current flowing through a solenoid produces magnetic field as shown in the figure. A close observation of magnetic lines of force reveal that they are parallel at the center of the solenoid and undergo fringing at the ends of the solenoid. Thus all magnetic lines of force (flux) produced by the solenoid link to its turns at the center of the solenoid. Whereas, linking of flux to turns of the solenoid near its ends is lesser.  Let, ϕ_k is the flux linking k^{th} turn when current i is flowing through the solenoid. magnetic flux ϕ_1. The flux produced by the solenoid and that linking turn of the solenoid can be determined applying Bio-Savart's Law. It is seen that the $\phi_k \propto i$...(1). The proportionality constant in (1) is related to **(a)** geometry of the solenoid and **(b)** position of the turn in the solenoid, as shown in the above diagram. Therefore, emf induced in a turn of a coil due to current in it as per Faraday's Law of Electromagnetic Induction, is $e \propto (-)\frac{d\phi}{dt} \Rightarrow e \propto (-)\frac{di}{dt} \Rightarrow e_k = K_k\frac{di}{dt}$ …(2). Equation (2) is expressed as $e_k = L_k\frac{di}{dt}$, where is called self-inductance of the k^{th} turn. Based on discussion and the diagram shown above, since flux linking turn in the center of the solenoid, due to current $i$flowing through the solenoid is greater than the turns nearing the end of the solenoid. Hence, **self-inductance of turn L_k for turn of the solenoid in the center of the coil is greater.**
Q-09	Consider the energy density in a solenoid at its center and that near its ends. Which of the two is greater?
I-09	Solenoid stores energy in the form of magnetic field. Solenoid stores magnetic energy when current flows through. Accordingly, energy density of a solenoid is quantified as magnetic energy per unit volume.

It is seen that when current flows through a wire, it produces magnetic field around itself. The wire is when bend to form a series of loops, it is called solenoid. A conceptual view of magnetic field produced by a solenoid is shown in the figure. A close observation of magnetic lines of force reveal that they are parallel at the center of the solenoid and undergo fringing at the ends of the solenoid. Thus all magnetic lines of force (flux) produced by the solenoid link to its turns at the center of the solenoid. Whereas, linking of flux to turns of the solenoid near its ends is lesser.



Let, ϕ_k is the flux linking k^{th} turn when current i is flowing through the solenoid. Magnetic flux ϕ_1. The flux produced by the solenoid and that linking turn of the solenoid can be determined applying Bio-Savart's Law. It is seen that the $\phi_k \propto i$…(1). The proportionality constant in (1) is related to **(a)** geometry of the solenoid and **(b)** position of the turn in the solenoid, as shown in the above diagram.

Therefore, emf induced in a turn of a coil due to current in it as per Faraday's Law of Electromagnetic Induction, is $e \propto (-)\frac{d\phi}{dt} \Rightarrow e \propto (-)\frac{di}{dt} \Rightarrow e_k = K_k\frac{di}{dt}$ …(2).

Equation (2) is expressed as $e_k = L_k\frac{di}{dt}$…(3), where is called self-inductance of the k^{th} turn. Here, L_k depends upon geometry and position of the turn in the solenoid. The ***L_k is maximum of the turns at the center of the solenoid.***

Instantaneous energy of the magnetic field associated with the kth turn is $U_{kt} = e_k i \Rightarrow U_{kt} = \left(L_k\frac{di}{dt}\right)i$. This can be written as $U_{kt} = L_k\left(i\frac{di}{dt}\right) \Rightarrow U_{kt}dt = L_k i di$. Thus, total energy stored in the turn of the solenoid is $U_{kt} = \int U_{kt}dt \Rightarrow U_k = L_k\int i di \Rightarrow U_k = L_k\left(\frac{1}{2}i^2\right) \Rightarrow U_k = \frac{1}{2}L_k i^2$. Since each turn of the solenoid has same width and same cross-sectional area. Therefore, volume of each turn is also constant say V. Therefore, energy density of kth turn, when current i is flowing through the solenoid is $\sigma_k = \frac{U_{kt}}{V} \Rightarrow \sigma_k = \frac{\frac{1}{2}L_k i^2}{V} \Rightarrow \sigma_k = \left(\frac{i^2}{2V}\right)L_k$ …(3)

Analyzing (3) in context of the discussion on inductance of kth turn of the solenoid is greater at the center. Therefore, **energy density of the turns in the center of the solenoid is greater.**

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