Let Us Do Some Problems-XXXXIII

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Some questions from the papers of entrance examinations for the admission to the undergraduate courses in *Mathematics*.

Q1. Define $a_n = (1^2 + 2^2 + \cdots + n^2)^n$ and $b_n = n^n(n!)^2$. Recall $n!$ is the product of the first n natural numbers. Then
(a)$a_n < b_n$ for all $n > 1$
(b)$a_n > b_n$ for all $n > 1$
(c)$a_n = b_n$ for infinitely many n
(d) none of the above
Ans.(b)

Q2. The last digit of $(2004)^5$ is
(a)4
(b)8
(c)6
(d)2
Ans.(a)

Q3. If n is positive integer such that $8n+1$ is a perfect square, then
(a)n must be odd
(b)n cannot be a perfect square
(c)$2n$ cannot be a perfect square
(d)none of the above
Ans.(c)

Q4. The coefficient of $a^3b^4c^5$ in the expansion of $(bc+ca+ab)^6$ is
(a)$\frac{12!}{3!4!5!}$
(b) $\frac{6!}{3!}$
(c)33
(d)$3.\frac{6!}{3!3!}$
Ans.(d)

Q5. If $\log_{10}(x) = 10^{\log_{10}4}$, then x equals to
(a)4^{10}
(b)100
(c)$\log_{10}4$
(d)none of the above
Ans.(b)

Q6. The set of all real numbers x such that $x^3(x+1)(x-2) \geq 0$ is
(a)the interval $2 \leq x < \infty$
(b)the interval $0 \leq x < \infty$
(c)the interval $-1 \leq x < \infty$
(d)none of the above
Ans.(d)

Q7. Let z be a non-zero complex number such that $\frac{z}{1+z}$ is purely imaginary then
(a) z is neither real nor purely imaginary
(b) z is real
(c) z is purely imaginary
(d) none of the above
Ans.(a)

Q8. In the interval $(0, 2\pi)$, the function $sin\left(\frac{1}{x^3}\right)$
(a) never changes sign
(b)changes sign only once
(c) changes sign more than once, but finitely many times
(d) changes sign infinitely many times
Ans.(d)

Q9.$\lim_{x\to 0}\frac{(e^x-1)tan^2x}{x^3}$
(a)does not exist
(b)exists and equals 0
(c)exists and equals 2/3
(d)exists and equals 1
Ans.(d)

Q10. If $f(x) = \cos(x) - 1 + \frac{x^2}{2}$, then
(a)*f(x)* is an increasing function on the real line
(b)*f(x)* is a decreasing function on the real line
(c) *f(x)* is increasing on the interval $-\infty < x \leq 0$ and decreasing on the interval $0 \leq x < \infty$
(d) *f(x)* is decreasing on the interval $-\infty < x \leq 0$ and increasing on the interval $0 \leq x < \infty$
Ans.(d)

Q11. The number of roots of the equation $x^2+\sin 2x=1$ in the closed interval $[0, \pi/2]$ is
(a)0
(b)1
(c)2
(d)3
Ans.(b)

Q12. The number of maps *f* from the set {1,2,3} into the set {1,2,3,4,5} such that $f(i) \leq f(j)$ whenever $i<j$ is
(a)60
(b)50
(c)35
(d)30
Ans. (c)

Q13. The digits in the unit place of the number 1!+2!+3!+….+99! is
(a)3
(b)0
(c)1
(d)7
Ans.(a)

Q14. The equation $x^3y+xy^3+xy=0$ represents
(a)a circle
(b)a circle and a pair of straight lines
(c)a rectangular hyperbola
(d)a pair of straight lines
Ans.(d)

Q15. For real numbers *x*, *y* and *z*, show that

$$|x| + |y| + |z| \leq |x + y - z| + |y + z - x| + |z + x - y|$$

Q16. How many real roots does $x^4 + 12x - 5$ have?
Ans.2

Q17. Consider the 2×2 matrix
$M = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \in M_2(R)$.
It the eighth power of M satisfies
$M^8 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$, then find the value of x.

Q18. Let *T* denote the sum of the convergent series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots. + \frac{(-1)^{n+1}}{n} + \cdots$$

And let *S* denote the sum of the convergent series

$$1 - \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{8} + \frac{1}{5} - \frac{1}{10} - \frac{1}{12} + \cdots = \sum_{n=1}^{\infty} a_n$$

Where $a_{2m-2} = \frac{1}{2m-1}, a_{2m-1} = \frac{-1}{4m-2}$ and $a_{2m} = \frac{-1}{4m}$ for m $\in N$, then find the relation between *T* and *S*.

Q19. Find the value of $\lim_{n\to\infty} \sum_{k=2}^{n} \frac{\sqrt{n+1}-\sqrt{n}}{k(\ln k)^2}$