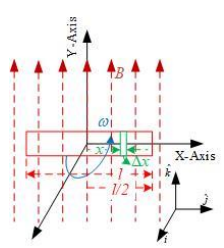
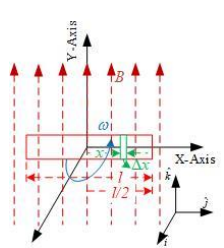
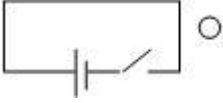
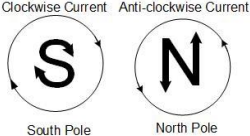
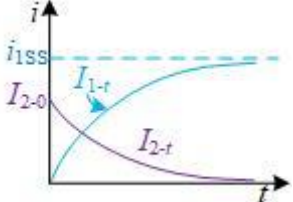
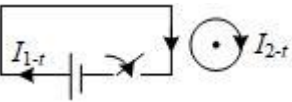
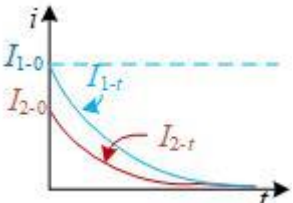


## Electromagnetism: Electromagnetic Induction [Part 3- Set 4(a-03)]

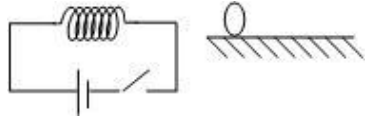
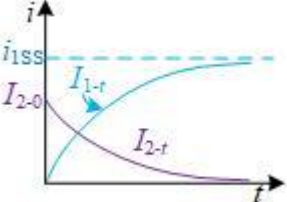
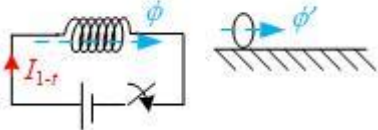
### Typical Questions with Illustrations

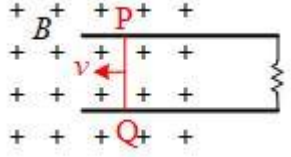
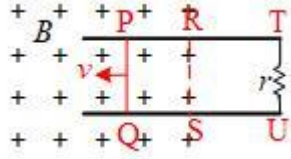
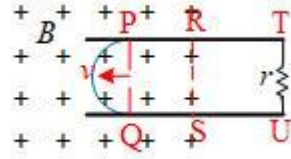
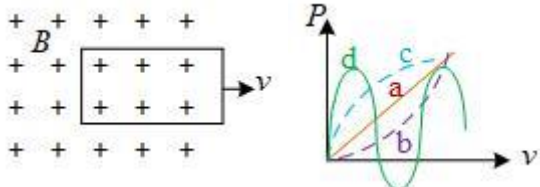
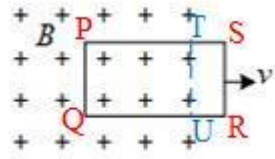
Q-01	<p>A metallic rod of length <math>l</math> rotates with a small but uniform angular velocity <math>\omega</math> about its perpendicular bisector. A uniform magnetic field <math>B</math> exists parallel to the axis of rotation. The potential difference between the center of the rod and an end is</p> <p>(a) Zero   (b) <math>\frac{1}{8}\omega Bl^2</math>   (c) <math>\frac{1}{2}\omega Bl^2</math>   (d) <math>\omega Bl^2</math></p>
I-01	<p>Given system is shown in the figure. For convenience of 3-D vector analysis of the electromagnetic phenomenon, unit direction vectors are also shown. At some instant the metallic rod of length <math>l</math>, along X-axis, has its center along Y-axis. The rod is rotating with an angular velocity <math>\vec{\omega} = \omega\hat{k}</math>. The space in which the rod is rotating has a uniform magnetic field <math>\vec{B} = B\hat{k}</math>.</p> <div style="text-align: right; margin-bottom: 10px;">  </div> <p>The metallic rod has free electron cloud, which in its state of rest, perform Brownian motion. When the rod is set in uniform angular motion <math>\vec{\omega} = \omega\hat{k}</math> the electron cloud is also set in motion.</p> <p>Let us consider a small elemental length <math>\Delta x</math>, at position <math>\vec{x} = x\hat{j}</math> from axis of rotation along <math>\hat{j}</math>. Therefore, its velocity <math>\vec{v} = \vec{\omega} \times \vec{x} \Rightarrow \vec{v} = (\omega\hat{k}) \times (x\hat{j}) \Rightarrow \vec{v} = \omega x(-\hat{i}) \dots (1)</math>.</p> <p>Number of electrons in the elemental length <math>\Delta q = n(-e)\Delta x \dots (2)</math>, here <math>n</math> is number of electrons per unit length of the rod and <math>(-e)</math> is the charge of an electron.</p> <p>Since the rod is rotating about <math>\hat{k}</math> in an external magnetic field <math>\vec{B}</math> only, while there is no external electric field, each electron would experience a electromagnetic force <math>\vec{F}_m</math>, in accordance with the Lorentz's Fore Law. Accordingly, <math>\Delta\vec{F}_m = (\Delta q)\vec{v} \times \vec{B} \Rightarrow \Delta\vec{F}_m = (-ne\Delta x)\vec{v} \times \vec{B} \dots (3)</math>.</p> <p>Thus, from (1), (2) and (3), we have <math>\vec{F}_m = (-ne\Delta x)(\omega x(-\hat{i})) \times (B\hat{k}) \Rightarrow \vec{F}_m = ne\omega Bx\Delta x(\hat{i} \times \hat{k}) \Rightarrow \vec{F}_m = ne\omega Bx\Delta x(-\hat{j}) \dots (3)</math>.</p> <p>The rod is an open circuit, despite <math>\vec{F}_m</math> the electron would not come out of the rod. This require some other force.</p> <p>Mass of electron is <math>m_e = 9.1 \times 10^{-31}\text{kg}</math> while charge of electron <math>e = (-)1.6 \times 10^{-19}</math>. It is seen that magnitude of mass is very small as comparted to charge of electron. While, centrifugal force <math>F_c = mr\omega^2</math>, which is sufficiently small and hence ignored in this problem.</p> <p>Therefore, the only force is electrostatic force <math>\vec{F}_E = q\vec{E} \Rightarrow \vec{F}_E = (-ne\Delta x)\vec{E} \dots (4)</math>, that would build is due to motion of electron under influence <math>\vec{F}_m</math> would build an equilibrium, which will not let the electron leave the rod as discussed due to the rod being open circuit. Accordingly, <math>\vec{F}_m + \vec{F}_E = 0 \Rightarrow \vec{F}_m = -\vec{F}_E \dots (5)</math>. Vectorially, this is possible only when magnitudes of the two vectors are equal and they are in opposite directions, as depicted by (-) sign in (5). Accordingly, <math>F_m = F_e \Rightarrow ne\omega Bx\Delta x = ne\Delta xE \Rightarrow \omega Bx\Delta x = E\Delta x \dots (6)</math>.</p> <p>Integrating (6) we get potential difference between end of the rod and axis of rotation, in this case as desired.</p> <p>Accordingly, <math>V = \int_0^{\frac{l}{2}} E dx = \int_0^{\frac{l}{2}} \omega Bx\Delta x \Rightarrow V = \omega B \left[ \frac{x^2}{2} \right]_0^{\frac{l}{2}} \Rightarrow V = \frac{\omega B}{2} \left[ \left( \frac{l}{2} \right)^2 \right] \Rightarrow V = \frac{\omega B}{2} \left( \frac{l^2}{4} \right) \Rightarrow V = \frac{\omega Bl^2}{8}</math>, this result is <b>provided in option (b)</b>. Hence, <b>answer is option (b)</b>.</p> <p><b>N.B.:</b> If we integrate (6) for the rod rotating about its perpendicular bisector, interval of the length is <math>\left(-\frac{l}{2}\right) \leq x \leq \frac{l}{2}</math>. This will lead to the result to be zero. If we compare this result from the Faraday's Law</p>

	<p>of electromagnetic induction <math>V = \frac{d}{dt} \phi</math>. Since, <math>\phi = \vec{A} \cdot \vec{B} \Rightarrow \phi = AB \cos \theta</math>. In this case <math>\theta = 0</math> since both <math>\vec{A}</math> and <math>\vec{B}</math> are collinear. Accordingly, <math>\cos \theta = 1</math> and <math>\phi = AB</math> a constant be it a loop or a rod of negligible width. This leads to <math>V = \frac{d}{dt} AB \Rightarrow V = 0</math>. Thus, prima facie, though this problem appears to be of Faraday's Laws of Electromagnetic Induction, actually it is application of Lorentz's Force Law.</p>
Q-02	<p>A rod of length <math>l</math> rotates with a small but uniform angular velocity <math>\omega</math> about its perpendicular bisector. A uniform magnetic field <math>B</math> exits parallel to the axis of rotation. The potential difference between the two ends of the rod is</p> <p>(a) Zero (b) <math>\frac{1}{2} \omega B l^2</math> (c) <math>\omega B l^2</math> (d) <math>2 \omega B l^2</math></p>
I-02	<p>Given system is shown in the figure. For convenience of 3-D vector analysis of the electromagnetic phenomenon, unit direction vectors are also shown. At some instant the metallic rod of length <math>l</math>, along X-axis, has its center along Y-axis. The rod is rotating with an angular velocity <math>\vec{\omega} = \omega \hat{k}</math>. The space in which the rod is rotating has a uniform magnetic field <math>\vec{B} = B \hat{k}</math>.</p>  <p>The metallic rod has free electron cloud, which in its state of rest, perform Brownian motion. When the rod is set in uniform angular motion <math>\vec{\omega} = \omega \hat{k}</math> the electron cloud is also set in motion.</p> <p>Let us consider a small elemental length <math>\Delta x</math>, at position <math>\vec{x} = x \hat{j}</math> from axis of rotation along <math>\hat{j}</math>. Therefore, its velocity <math>\vec{v} = \vec{\omega} \times \vec{x} \Rightarrow \vec{v} = (\omega \hat{k}) \times (x \hat{j}) \Rightarrow \vec{v} = \omega x (-\hat{i}) \dots (1)</math>.</p> <p>Number of electrons in the elemental length <math>\Delta q = n(-e) \Delta x \dots (2)</math>, here <math>n</math> is number of electrons per unit length of the rod and <math>(-e)</math> is the charge of an electron.</p> <p>Since the rod is rotating about <math>\hat{k}</math> in an external magnetic field <math>\vec{B}</math> only, while there is no external electric field, each electron would experience a electromagnetic force <math>\vec{F}_m</math>, in accordance with the Lorentz's Fore Law. Accordingly, <math>\Delta \vec{F}_m = (\Delta q) \vec{v} \times \vec{B} \Rightarrow \Delta \vec{F}_m = (-ne \Delta x) \vec{v} \times \vec{B} \dots (3)</math>.</p> <p>Thus, from (1), (2) and (3), we have <math>\vec{F}_m = (-ne \Delta x) (\omega x (-\hat{i})) \times (B \hat{k}) \Rightarrow \vec{F}_m = ne \omega B x \Delta x (\hat{i} \times \hat{k}) \Rightarrow \vec{F}_m = ne \omega B x \Delta x (-\hat{j}) \dots (3)</math>.</p> <p>The rod is an open circuit, despite <math>\vec{F}_m</math> the electron would not come out of the rod. This require some other force.</p> <p>Mass of electron is <math>m_e = 9.1 \times 10^{-31} \text{kg}</math> while charge of electron <math>e = (-)1.6 \times 10^{-19}</math>. It is seen that magnitude of mass is very small as comparted to charge of electron. While, centrifugal force <math>F_c = mr\omega^2</math>, which is sufficiently small and hence ignored in this problem.</p> <p>Therefore, the only force is electrostatic force <math>\vec{F}_E = q\vec{E} \Rightarrow \vec{F}_E = (-ne \Delta x) \vec{E} \dots (4)</math>, that would build is due to motion of electron under influence <math>\vec{F}_m</math> would build an equilibrium, which will not let the electron leave the rod as discussed due to the rod being open circuit. Accordingly, <math>\vec{F}_m + \vec{F}_E = 0 \Rightarrow \vec{F}_m = -\vec{F}_E \dots (5)</math>. Vectorially, this is possible only when magnitudes of the two vectors are equal and they are in opposite directions, as depicted by (-) sign in (5). Accordingly, <math>F_m = F_e \Rightarrow ne \omega B x \Delta x = ne \Delta x E \Rightarrow \omega B x \Delta x = E \Delta x \dots (6)</math>.</p> <p>Integrating (6) we get potential difference between ends of the rod, as desired. Accordingly, <math>V = \int_{-\frac{l}{2}}^{\frac{l}{2}} E dx = \int_{-\frac{l}{2}}^{\frac{l}{2}} \omega B x \Delta x \Rightarrow V = \omega B \left[ \frac{x^2}{2} \right]_{-\frac{l}{2}}^{\frac{l}{2}} \Rightarrow V = \frac{\omega B}{2} \left[ \left( \frac{l}{2} \right)^2 - \left( -\frac{l}{2} \right)^2 \right] \Rightarrow V = \frac{\omega B}{2} \left( \frac{l^2}{4} - \frac{l^2}{4} \right) \Rightarrow V = 0</math>, this result is <b>provided in option (a). Hence, answer is option (a).</b></p> <p><b>N.B.:</b> As per Faraday's Law of Electromagnetic Induction we know <math>V = \frac{d}{dt} \phi</math>. Since, <math>\phi = \vec{A} \cdot \vec{B} \Rightarrow \phi = AB \cos \theta</math>. In this case <math>\theta = 0</math> since both <math>\vec{A}</math> and <math>\vec{B}</math> are collinear. Accordingly, <math>\cos \theta = 1</math> and <math>\phi = AB</math> a</p>

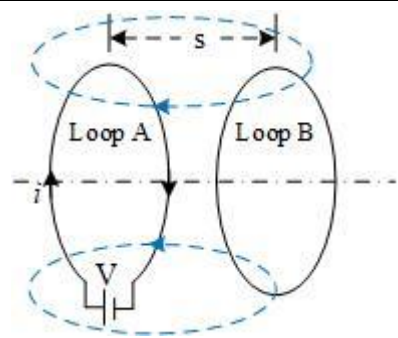
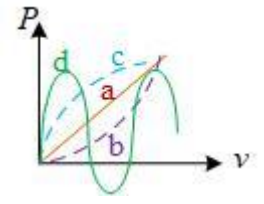
	<p>constant be it a loop or a rod of negligible width. This leads to <math>V = \frac{d}{dt} AB \Rightarrow V = 0</math>. Prima facie, though this problem appears to be of Faraday's Laws of Electromagnetic Induction, actually it is application of Lorentz's Force Law.</p>
<p>Q-03</p>	<p>Consider the situation shown in the figure. If the switch is closed and after some time it is opened again, the closed loop will show</p>  <p>(a) An anticlockwise current-pulse  (b) A clockwise current-pulse  (c) An anticlockwise current-pulse and then a clockwise current-pulse  (d) A clockwise current-pulse and then an anticlockwise current-pulse</p>
<p>I-03</p>	<p>When switch in the circuit is closed it will establish a clockwise current in the circuit as shown in the figure. As a result a magnetic field would be established around the wire forming the circuit. Accordingly, as per Ampere's Right-Hand Screw Rule (ARHSR) the magnetic field produced by the current in the circuit will come out the loop as shown in the figure.</p>  <p>Now when switch is closed, current <math>I_{1-t}</math> in the circuit will build up exponentially until it reaches steady state value <math>I_{1SS}</math>, as shown in the figure. This current <math>I_{1-t}</math> will induce instantaneous flux out of which <math>\phi_{2t} \propto I_{1-t}</math> mutually links to the loop. This in-turn will induce emf <math>E_t = \frac{d}{dt} \phi_{2t}</math> in loop, as per Faraday's Law of Electromagnetic Induction (FLEI).</p>  <p>This induced <math>E_t</math> will cause flow of current <math>I_{2-t} = \frac{E_t}{R}</math> in the loop in accordance with Ohm's Law (OL). <math>I_{2-t} = \frac{E_t}{R}</math>, here <math>R</math> is the resistance of the loop. The current in <math>I_{2-t}</math> will also decay as shown in the figure.</p> <p>Thus is as per ARHSR, FLEI and OL will produce flux – (a) coming out of the face of figure, and (b) increasing exponentially.</p> <p>Now comes Lenz's Law (LL) according to which direction of induced emf, and in turn induced current <math>I_{2-t}</math>, is such that it opposes cause of its production. The cause of production in the instant case is <math>I_{1-t}</math>. Accordingly, <math>\phi_t'</math> produced by current induced in the loop <math>I_{2-t}</math> will be tending to oppose reducing <math>\phi_{2t}</math>. This is possible only when <math>\phi_t'</math> increases when <math>\phi_{2t}</math> decreases. While this happen rate of changes <math>\frac{d}{dt} \phi_t' = -\frac{d}{dt} \phi_{2t}</math>, but the direction of flux remains same. This leads to the conclusion that <b>when the switch is closed current pulse induced in the loop is clockwise.</b></p>  <p>But when switch is opened, the current the circuit will start decaying as shown in the and hence just a reverse of the process, that occurred on closing of the switch will occur. Therefore, again as per ARHSR, when switch is opened <b>current pulse induced in the loop is anticlockwise.</b></p>  <p>These conclusions are provided in option (d). Hence, the <b>answer is option (d)</b></p>
<p>Q-04</p>	<p>Consider the situation shown in the figure, it's closed loop is completely enclosed in the circuit containing the switch. If the switch is closed and after some time it is opened again, the closed loop will show</p> <p>(a) An anticlockwise current-pulse  (b) A clockwise current-pulse  (c) An anticlockwise current-pulse and then a clockwise current-pulse  (d) A clockwise current-pulse and then an anticlockwise current-pulse</p>

<p>I-04</p>	<p>When switch in the circuit is closed it will establish a clockwise current in the circuit as shown in the figure. As a result a magnetic field would be established around the wire forming the circuit. Accordingly, as per Ampere's Right-Hand Screw Rule (ARHSR) the magnetic field produced by the current in the circuit will come out the loop as shown in the figure.</p> <p>Now when switch is closed, current <math>I_{1-t}</math> in the circuit will build up exponentially until it reaches steady state value <math>I_{1SS}</math>, as shown in the figure. This current <math>I_{1-t}</math> will induce instantaneous flux out of which <math>\phi_{2t} \propto I_{1-t}</math> mutually links to the loop. This in-turn will induce emf <math>E_t = \frac{d}{dt} \phi_{2t}</math> in loop, as per Faraday's Law of Electromagnetic Induction (FLEI).</p> <p>This induced <math>E_t</math> will cause flow of current <math>I_{2-t} = \frac{E_t}{R}</math> in the loop in accordance with Ohm's Law (OL). <math>I_{2-t} = \frac{E_t}{R}</math>, here <math>R</math> is the resistance of the loop. The current in <math>I_{2-t}</math> will also decay as shown in the figure.</p> <p>Thus is as per ARHSR, FLEI and OL will produce flux – (a) coming out of the face of figure, and (b) increasing exponentially.</p> <p>Now comes Lenz's Law (LL) according to which direction of induced emf, and in turn induced current <math>I_{2-t}</math>, is such that it opposes cause of its production. The cause of production in the instant case is <math>I_{1-t}</math>. Accordingly, <math>\phi_t'</math> produced by current induced in the loop <math>I_{2-t}</math> will be tending to oppose reducing <math>\phi_{2t}</math>. This is possible only when <math>\phi_t'</math> increases when <math>\phi_{2t}</math> decreases. While this happen rate of changes <math>\frac{d}{dt} \phi_t' = -\frac{d}{dt} \phi_{2t}</math>, but the direction of flux remains same. This leads to the conclusion that <b>when the switch is closed current pulse induced in the loop is anticlockwise.</b></p> <p>But when switch is opened, the current the circuit will start decaying as shown in the and hence just a reverse of the process, that occurred on closing of the switch will occur. Therefore, again as per ARHSR, when switch is opened <b>current pulse induced in the loop is clockwise.</b></p> <p>These conclusions are provided in option (d). Hence, the <b>answer is option (c)</b></p>
<p>Q-05</p>	<p>A bar magnet is released from rest along the axis of a very long vertical copper tube. After sometime the magnet</p> <p>(a) Will stop in the tube  (b) Will move with almost constant speed  (c) Will move with an acceleration <math>g</math>  (d) Will oscillate</p>
<p>I-05</p>	<p>Given that a bar magnet is released from rest (initial velocity, <math>u = 0</math>) inside a copper tube as shown in the figure.</p> <p>The system experiences three phenomenon –</p> <p>(a) Magnet while falling, if there were no copper tube around it, would experience a constant acceleration due to gravity (<math>g</math>). Accordingly, its velocity of fall would tend to increase as per Third equation of motion <math>v^2 = u^2 + 2gd \dots (1)</math>. Here, <math>d</math> is the depth of fall from initial position when <math>u = 0</math>, and <math>v</math> is the velocity of the magnet at depth <math>d</math>; it tend to increase as it falls progressively.</p> <p>(b) Gravitational force experienced by the bar magnet is <math>F_g = mg \dots (2)</math>, here <math>m</math> is mass of the bar magnet. Combining (1) and (2), <math>F_g \propto m \frac{v^2}{d} \Rightarrow F_g \propto \frac{v^2}{d} \dots (3)</math></p> <p>(c) The bar magnet produces magnetic field which intercepts surface of the tube. Intensity of the magnetic field decreases with distance of the point of surface of tube from the magnet.</p>

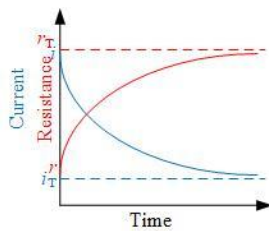
	<p>(d) Thus, when magnet falls each point on the surface of the tube will experience rate of change of magnetic field as the magnets passes by.</p> <p>(e) The phenomenon at (c) would produce eddy currents on the surface of the tube. The magnitude of eddy currents (<math>i_e</math>), as per Faraday's Law and Ohm's Law, would be proportional to velocity i.e. <math>i_e \propto v \dots(4)</math>.</p> <p>(f) Direction of eddy current (<math>i_e</math>), as per Lenz's Law, is to oppose cause of production i.e. velocity of fall (<math>v</math>).</p> <p>(g) Thus, eddy currents in presence of magnetic field, as per Lorentz's Force Law would create an electromagnetic force <math>F_E \propto v \dots(5)</math>, in a direction opposite to constant gravitational force <math>F_g</math>. Thus, <math>F_E</math> would retard velocity of fall. Yet, buildup of velocity of fall would continue until an equilibrium is reached where <math>F_E = F_g \dots(6)</math>.</p> <p>(h) Combining (3) and (5) in (6), equilibrium reached after a fall through depth <math>D</math> we have <math>\frac{v^2}{D} \propto v</math>. It leads to <math>v \propto D \dots(7)</math>.</p> <p>Above discussions leads conclusion that after some time after falling through a certain height, the bar magnet will move continue to fall with a constant velocity, as provided in option (b). Hence, <b>answer is option (d)</b>.</p>
Q-06	<p>Figure shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed the ring will</p> <p>(a) Remain stationary          (b) Move towards the solenoid          (c) Move away from the solenoid          (d) Move towards or away from the solenoid depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.</p> 
I-06	<p>When switch is closed, it will build current <math>i_{1-t}</math> in the circuit, as shown in the figure. The buildup of the current <math>i_{1-t}</math> is exponential as shown in the figure below. Accordingly, the solenoid would build a magnetic flux <math>\phi</math>, applying Biot-Savart's Law. Out of the flux created by the solenoid, a part of the flux <math>\phi' \propto \phi \dots(1)</math>, would mutually link the copper ring placed coaxially on a frictionless surface.</p>  <p>Since, <math>\phi \propto i_{1-t} \Rightarrow \phi' \propto i_{1-t} \dots(2)</math>, and it is time variant, therefore as per Faraday's law of electromagnetic induction combined with Ohm's Law would produce time varying current <math>i_{rt} \propto \frac{d}{dt} \phi' \Rightarrow i_{rt} \propto \frac{d}{dt} \dots(3)</math>. This current <math>i_{rt}</math> in the ring would produce induced flux <math>\phi''</math> which is in direction such that it opposes <math>\phi</math> as per Lenz's Law.</p> <p>Accordingly, magnetic polarity of <math>\phi</math> and <math>\phi''</math> would be like either both are N pole or S pole. This, would cause a force of repulsion in like poles, leading to the ring moving away from the coil, as given in option (c). Hence, <b>answer is option (c)</b>.</p> 
Q-07	<p>Consider the following statements:</p> <p>A) An emf can be induced by moving a conductor in a magnetic field          B) An emf can be induced by changing the magnetic field</p> <p>a) Both A and B are true (b) A is true but B is false (c) B is true but A is false (d) Both A and B are false</p>
I-07	<p>It requires to analyze each statement for validity, separately. It is essential to choose among the given options based on validity of the two statements. Accordingly –</p> <p><b>Statement A:</b> Emf <math>\vec{E}</math> induced along a conductor of length <math>\vec{l} = l\hat{l}</math> intercepting magnetic field <math>\vec{B}</math> with a velocity <math>\vec{v}</math> is <math>\vec{E} = (\vec{B} \times \vec{v})l \Rightarrow \vec{E} = Bvl\hat{l}</math>, magnitude of emf along is <math>E = Bvl</math>. Thus, <b>option (A) is correct</b>.</p>

	<p><b>Statement B:</b> A conductor has a cross-section, which can be considered to be a spiral-coil of thin strips. Flux <math>\phi</math> intercepting the spiral coil is changing. Hence as per Faraday's Law of electromagnetic induction <math>E = \frac{d}{dx} \phi</math>. Accordingly, statement B is also true.</p> <p>Both the given statements are true as provided in option (a), <b>hence answer is option (a)</b></p>
Q-08	<p>Consider the situation shown in figure. The wire AB is slid on the fixed rail with a constant velocity. If the wire AB is replaced by a semicircular wire. The magnitude of the induced current will</p> <p>(a) Increase (b) Remain the same (c) Decrease</p> <p>Increase or decrease depending on whether the semi-circle bulges towards the resistance or away from it</p> 
I-08	<p>Consider the given system where wire PQ together with the rails and resistance connected to the ends of the rail form a loop P-R-T-U-S-Q. Conceptually, area of this loop is <math>PRTUSQ_{area} = PRSQ_{area} + RTUS_{area}</math>. Yet, magnetic field <math>B</math> exists only in area <math>A = PRSQ_{area} \dots (1)</math>. Therefore, flux linking the loop is <math>\phi = B(A) \dots (2)</math>.</p> <p>Now, the wire PQ of length <math>l</math> is sliding on the rail with a velocity <math>v</math>, and the wire remain inside the magnetic field <math>B</math>. In that case rate of change of area of the loop <math>PRSQ_{area}</math> is <math>\frac{d}{dt} A = lv \dots (3)</math>. Therefore rate of change of flux-linkage <math>\frac{d}{dt} \phi = B \left( \frac{d}{dt} A \right) \Rightarrow \frac{d}{dt} \phi = Blv \dots (4)</math>. As per Faraday's Law of Electromagnetic Induction emf in the loop PRTUSQ is <math>e = \frac{d}{dt} \phi \Rightarrow e = Blv \dots (5)</math>. Hence, as per Ohm's Law current in the loop having resistance <math>r</math> is <math>I = \frac{e}{r} \Rightarrow I = \frac{Blv}{r} \dots (6)</math>.</p> <p>In next stage wire PQ is replaced with a semicircular wire as shown in the figure. Therefore flux linking will be <math>\phi' = BA'</math>, here <math>A' = \left( A + \frac{\pi l^2}{8} \right) \dots (7)</math>, is the area of the of the loop, of which <math>A_s = \frac{\pi l^2}{8}</math> is the area of the semicircular part of the loop.</p> <p>When, the wire, in this case semicircular arc, slides with velocity <math>v</math>, area <math>A_s</math> remains constant. Therefore, rate of change of area, while the wire slides is <math>\frac{d}{dt} A' = \frac{d}{dt} \left( A + \frac{\pi l^2}{8} \right) \Rightarrow \frac{d}{dt} A' = \frac{d}{dt} A + \frac{d}{dt} \left( \frac{\pi l^2}{8} \right) \Rightarrow \frac{d}{dt} A' = \frac{d}{dt} A \dots (8)</math>. Accordingly, combining(4), (5) and (8), emf induced due to sliding of the semicircular wire is <math>e' = Blv \dots (9)</math>. Since, resistance of the loop remains same at <math>r</math>, hence current in the loop, as per Ohm's Law is <math>I' = \frac{e'}{r} \Rightarrow I' = \frac{Blv}{r} \dots (10)</math>.</p> <p>Comparing results at (6) ad (10), we find current in the loop, in both the cases, is <math>I = I' = \frac{Blv}{r}</math>, as provided in the option (b). Hence, <b>answer is option (c)</b>.</p>  
Q-09	<p>Figure (a) shows a conducting loop being pulled out a magnetic field with a speed <math>v</math>. Which of the four plots shown in the figure (b) may represent the power delivered by the pulling agent as a function of the speed <math>v</math>?</p> 
I-09	<p>The given system is elaborated in figure. The rectangular conducting loop PQRS of area <math>A</math>, with an imaginary line TU, is divided in two loops PQUT of area <math>A'</math> and TURS of area <math>A''</math>. The area <math>A'</math> is perpendicular to magnetic field <math>B</math>, while there is no magnetic field intercepting area <math>A''</math>.</p> <p>When, the conducting loop having an inherent resistance <math>r</math>, is being pulled with a velocity <math>v</math>, without change in shape of the loop i.e. <math>\frac{d}{dt} A = 0 \dots (1)</math> This pulling of</p> 

	<p>loop causes edge PQ and SR moving towards right with velocity <math>v</math>, while side TU remains fixed o the boundary of static magnetic field. Accordingly, <math>\frac{d}{dt} A = \frac{d}{dt} (A' + A'') \Rightarrow \frac{d}{dt} A = \frac{d}{dt} A' + \frac{d}{dt} A'' \dots(2)</math>.</p> <p>Combining (1) and (2) leads to <math>\frac{d}{dt} A' + \frac{d}{dt} A'' = 0 \Rightarrow \frac{d}{dt} A'' = -\frac{d}{dt} A' = lv \dots(3)</math>. Here, <math>l</math> is length of side PQ.</p> <p>It is seen form the figure that when the loop is being pulled out of the magnetic field, <math>\frac{d}{dt} A'' &gt; 0</math> and <math>\frac{d}{dt} A' &lt; 0</math>. Therefore, rate of change of flux linkage in the loop PQRS, to which magnetic field is <math>\perp</math> to its surface is <math>\frac{d}{dt} \phi = B \left( \frac{d}{dt} A' \right) \dots(4)</math>.</p> <p>Therefore, emf induced in the loop is <math>e = -\frac{d}{dt} \phi \dots(5)</math>. Combining (3), (4) and (5), <math>e = -B(-lv) \Rightarrow e = Blv \dots(6)</math>.</p> <p>Power required for the motion of loop is <math>P = ei \Rightarrow P = \frac{e^2}{r} \Rightarrow P \propto e^2 \dots(7)</math>.</p> <p>Observation of the curves given in the option are as under –</p> <p><b>Curve (a)</b> – It is linearly increasing and does not comply with quadratic nature in (7). Hence, <b>it is incorrect</b>.</p> <p><b>Curve (b)</b> – It is an increasing curve but of quadratic nature and conforms to (7). Hence, <b>it is correct</b>,</p> <p><b>Curve (c)</b>: It is an exponentially increasing curve and does not conform to (7). Hence, <b>it is incorrect</b>.</p> <p><b>Curve (d)</b>: It is a sinusoidal curves and does not conform to (7). Hence, <b>it is incorrect</b>.</p> <p>Thus, <b>answer is option (b)</b>.</p>
Q-10	<p>Two circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops</p> <p>(a) Attract each other  (b) Repel each other  (c) Do not exert any force on each other  (d) Attract or repel each other depending on the sense of current</p>
I-10	<p>Given are two loops A and B are identical geometry are placed coaxially with a seperation say <math>s</math>. Say loop A is cut a battery of <math>V</math> volts is inserted as shown in the figure. Let resistances of the loops are <math>r</math>.</p> <p>When battery comes in circuit, in accordance with the Ohm's Law, a current <math>i = \frac{V}{r}</math> would be established. Further, the current in the loop would produce a flux <math>\phi</math> part of it <math>\phi'</math> would be linking loop B.</p> <p>The current would produce heat <math>H = i^2 r</math> due to losses occuring in the loop A. This, heat <math>H</math> would increase temperature of the loop from ambient <math>T_a</math> to a steadystate temperature <math>T_{ss}</math>.</p>



During the process of reaching steady state resistance of loop would increase to  $r_T = r(1 + \alpha(T_{ss} - T_a))$  and also heat develop in the loop to  $H_T = i^2 r_T$ . As a result of increase in resistance current in the loop would decrease from  $i$  to  $i_T = \frac{V}{r_T}$ .

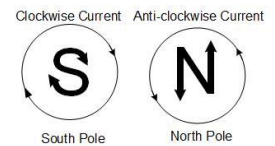


Qualitatively, change in resistance of loop A and current in it, is conceptualized in figure, until heat radiated by loop is equal to  $H$ .

This variation of decreasing current  $\frac{d}{dt}i < 0$  would produce a time varying flux  $\frac{d}{dt}\phi < 0$  linking loop A and accordingly time-varying flux  $\frac{d}{dt}\phi' < 0$  linking B.

As per Lenz's Law  $\frac{d}{dt}\phi'$  would produce an emf  $e_B$  in Loop B to oppose the reduction of flux  $\phi'$  and in turn primary cause reducing flux  $\phi$  linking Loop A. *It implies that current in Loop B reinforces  $\phi$ . This is possible only when magnetic polarity of loop B is opposite to that of the loop A.*

Applying Ampere's Righthand Thumb Rule, magnetic polarity of loop A due to current caused in it by battery is S. Therefore, magnetic polarity of Loop B due to current induced in it N.



Opposite magnetic polarities produce a force of attraction. Accordingly, during the period of decay of current in Loop, the two loop would attract each other, as provided in Option (a), irrespective of the direction of current in loop or polarity of battery inserted in it.

Hence, **answer is option (a).**